3. Using the given conversion factors, we find

(a) the distance *d in rods* to be

$$d = 4.0 \text{ furlongs} = \frac{(4.0 \text{ furlongs})(201.168 \text{ m/furlong})}{5.0292 \text{ m/rod}} = 160 \text{ rods},$$

(b) and that distance *in chains* to be

$$d = \frac{(4.0 \text{ furlongs})(201.168 \text{ m/furlong})}{20.117 \text{ m/chain}} = 40 \text{ chains}.$$

5. Various geometric formulas are given in Appendix E.

(a) Expressing the radius of the Earth as

$$R = \left(6.37 \times 10^6 \text{ m}\right)\left(10^{-3} \text{ km/m}\right) = 6.37 \times 10^3 \text{ km},$$

its circumference is $s = 2\pi R = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}$.

(b) The surface area of Earth is $A = 4\pi R^2 = 4\pi \left(6.37 \times 10^3 \text{ km}\right)^2 = 5.10 \times 10^8 \text{ km}^2$.

(c) The volume of Earth is $V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} \left(6.37 \times 10^3 \text{ km}\right)^3 = 1.08 \times 10^{12} \text{ km}^3$.

17. None of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important is that the clock advance by the same amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval. If the clock reading jumps around from one 24-h period to another, it cannot be corrected since it would impossible to tell what the correction should be. The following gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat.
A	–16	–16	–15	–17	–15	–15
B	–3	+5	–10	+5	+6	–7
C	–58	–58	–58	–58	–58	–58
D	+67	+67	+67	+67	+67	+67

E	+70	+55	+2	+20	+10	+10

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made "perfect" with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from -5 s to +10 s, for clock E it is in the range from -70 s to -2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

21. We introduce the notion of density:

$$\rho = \frac{m}{V}$$

and convert to SI units: 1 g = 1×10^{-3} kg.

(a) For volume conversion, we find 1 cm^3 = $(1 \times 10^{-2}\text{m})^3 = 1 \times 10^{-6}\text{m}^3$. Thus, the density in kg/m^3 is

$$1\ \text{g/cm}^3 = \left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\ \text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\ \text{m}^3}\right) = 1 \times 10^3\ \text{kg/m}^3.$$

Thus, the mass of a cubic meter of water is 1000 kg.

(b) We divide the mass of the water by the time taken to drain it. The mass is found from $M = \rho V$ (the product of the volume of water and its density):

$$M = \left(5700\ \text{m}^3\right)\left(1 \times 10^3\ \text{kg/m}^3\right) = 5.70 \times 10^6\ \text{kg}.$$

The time is t = (10h)(3600 s/h) = 3.6×10^4 s, so the *mass flow rate R* is

$$R = \frac{M}{t} = \frac{5.70 \times 10^6\ \text{kg}}{3.6 \times 10^4\ \text{s}} = 158\ \text{kg/s}.$$

35. (a) Dividing 750 miles by the expected "40 miles per gallon" leads the tourist to believe that the car should need 18.8 gallons (in the U.S.) for the trip.

(b) Dividing the two numbers given (to high precision) in the problem (and rounding off) gives the conversion between U.K. and U.S. gallons. The U.K. gallon is larger than the U.S gallon by a factor of 1.2. Applying this to the result of part (a), we find the answer for part (b) is 22.5 gallons.

39. Using the (exact) conversion 2.54 cm = 1 in. we find that 1 ft = (12)(2.54)/100 = 0.3048 m (which also can be found in Appendix D). The volume of a cord of wood is $8 \times 4 \times 4 = 128\ \text{ft}^3$, which we convert (multiplying by 0.3048^3) to $3.6\ \text{m}^3$. Therefore, one cubic meter of wood corresponds to $1/3.6 \approx 0.3$ cord.

41. (a) The difference between the total amounts in "freight" and "displacement" tons, (8 − 7)(73) = 73 barrels bulk, represents the extra M&M's that are shipped. Using the conversions in the problem, this is equivalent to (73)(0.1415)(28.378) = 293 U.S. bushels.

(b) The difference between the total amounts in "register" and "displacement" tons, (20 − 7)(73) = 949 barrels bulk, represents the extra M&M's are shipped. Using the conversions in the problem, this is equivalent to (949)(0.1415)(28.378) = 3.81×10^3 U.S. bushels.

45. We convert meters to astronomical units, and seconds to minutes, using

$$\begin{aligned} 1000\ \text{m} &= 1\ \text{km} \\ 1\ \text{AU} &= 1.50 \times 10^8\ \text{km} \\ 60\ \text{s} &= 1\ \text{min}. \end{aligned}$$

Thus, 3.0×10^8 m/s becomes

$$\left(\frac{3.0 \times 10^8\ \text{m}}{\text{s}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{\text{AU}}{1.50 \times 10^8\ \text{km}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = 0.12\ \text{AU/min}.$$

57. (a) When θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ, such as we deal with in Fig. 1–9, the arc may be approximated as the straight line-segment of length 1 AU. First, we convert θ = 1 arcsecond to radians:

$$(1\ \text{arcsecond})\left(\frac{1\ \text{arcminute}}{60\ \text{arcsecond}}\right)\left(\frac{1^\circ}{60\ \text{arcminute}}\right)\left(\frac{2\pi\ \text{radian}}{360^\circ}\right)$$

which yields $\theta = 4.85 \times 10^{-6}$ rad. Therefore, one parsec is

$$R_o = \frac{s}{\theta} = \frac{1\ \text{AU}}{4.85 \times 10^{-6}} = 2.06 \times 10^5\ \text{AU}.$$

Now we use this to convert R = 1 AU to parsecs:

$$R = (1\ \text{AU})\left(\frac{1\ \text{pc}}{2.06 \times 10^5\ \text{AU}}\right) = 4.9 \times 10^{-6}\ \text{pc}.$$

(b) Also, since it is straightforward to figure the number of seconds in a year (about 3.16×10^7 s), and (for constant speeds) distance = speed × time, we have

$$1\,\text{ly} = (186{,}000\,\text{mi/s})\,(3.16 \times 10^7\ \text{s})\ 5.9 \times 10^{12}\ \text{mi}$$

which we convert to AU by dividing by 92.6×10^6 (given in the problem statement), obtaining 6.3×10^4 AU. Inverting, the result is 1 AU = $1/6.3 \times 10^4 = 1.6 \times 10^{-5}$ ly.

Chapter 2 – Student Solutions Manual

1. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\, t_c$.

(a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time interval is

$$t_1 = \frac{(40\ \text{km})}{(30\ \text{km/h})} = 1.33\ \text{h}.$$

During the second part the displacement is $\Delta x_2 = 40$ km and the time interval is

$$t_2 = \frac{(40\ \text{km})}{(60\ \text{km/h})} = 0.67\ \text{h}.$$

Both displacements are in the same direction, so the total displacement is

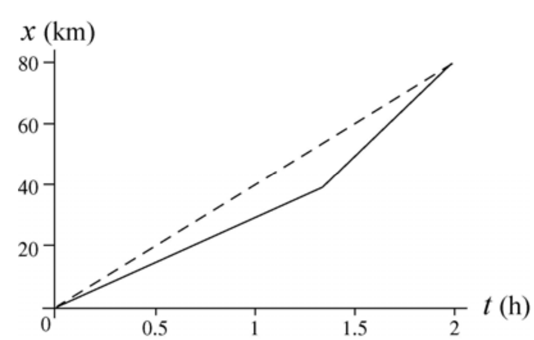
$$\Delta x = \Delta x_1 + \Delta x_2 = 40\ \text{km} + 40\ \text{km} = 80\ \text{km}.$$

The total time for the trip is $t = t_1 + t_2 = 2.00$ h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{(80\ \text{km})}{(2.0\ \text{h})} = 40\ \text{km/h}.$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(t_1, x_1) = (1.33$ h, 40 km) and the second having a slope of 60 km/h and connecting (t_1, x_1) to $(t, x) = (2.00$ h, 80 km). From the graphical point of view, the slope of the dashed line drawn from the origin to (t, x) represents the average velocity.



5. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write

$$x = (3\ \text{m/s})t - (4\ \text{m/s}^2)t^2 + (1\ \text{m/s}^3)t^3.$$

We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in $t = 1$ s yields $x = 3 - 4 + 1 = 0$.

(b) With $t = 2$ s we get $x = 3(2) - 4(2)^2 + (2)^3 = -2$ m.

(c) With $t = 3$ s we have $x = 0$ m.

(d) Plugging in $t = 4$ s gives $x = 12$ m.

For later reference, we also note that the position at $t = 0$ is $x = 0$.

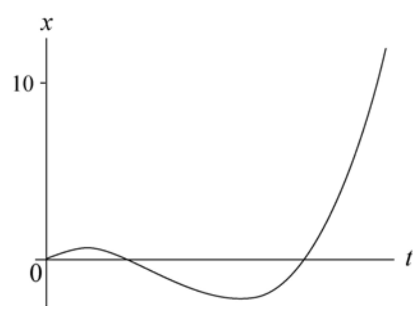
(e) The position at $t = 0$ is subtracted from the position at $t = 4$ s to find the displacement $\Delta x = 12$ m.

(f) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement $\Delta x = 14$ m. Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\ \text{m}}{2\ \text{s}} = 7\ \text{m/s}.$$

(g) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2, -2)$ to the highest point shown (at $t = 4$ s) which would represent the answer for part (f).



19. We represent its initial direction of motion as the $+x$ direction, so that $v_0 = +18$ m/s and $v = -30$ m/s (when $t = 2.4$ s). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30\text{ m/s}) - (+1\text{ m/s})}{2.4\text{ s}} = -20\text{ m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

25. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}(3.0\times 10^8\text{ m/s})}{9.8\text{ m/s}^2} = 3.1\times 10^6\text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8\text{ m/s}^2\right)(3.1\times 10^6\text{ s})^2 = 4.6\times 10^{13}\text{ m}.$$

27. Assuming constant acceleration permits the use of the equations in Table 2-1. We solve $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $x = 0.010$ m. Thus,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times 10^5\text{ m/s})^2 - (1.5\times 10^5\text{ m/s})^2}{2(0.010\text{ m})} = 1.62\times 10^{15}\text{ m/s}^2.$$

33. The problem statement (see part (a)) indicates that a = constant, which allows us to use Table 2-1.

(a) We take $x_0 = 0$, and solve $x = v_0t + \frac{1}{2}at^2$ (Eq. 2-15) for the acceleration: $a = 2(x - v_0t)/t^2$. Substituting $x = 24.0$ m, $v_0 = 56.0$ km/h $= 15.55$ m/s and $t = 2.00$ s, we find

$$a = \frac{2\left(24.0\text{m} - (15.55\text{m/s})\ (2.00\text{s})\right)}{(2.00\text{s})^2} = -3.56\text{m/s}^2,$$

or $|a| = 3.56\text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate $v = v_0 + at$ as follows:

$$v = 15.55\ \text{m/s} - \left(3.56\ \text{m/s}^2\right)\left(2.00\ \text{s}\right) = 8.43\ \text{m/s}$$

which can also be converted to 30.3 km/h.

45. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Starting the clock at the moment the wrench is dropped ($v_0 = 0$), then $v^2 = v_o^2 - 2g\Delta y$ leads to

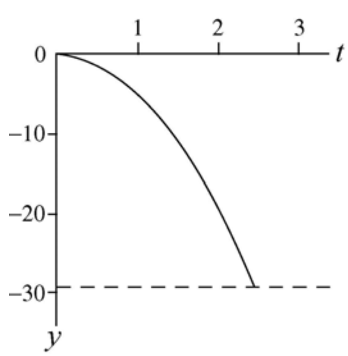
$$\Delta y = -\frac{(-24\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = -29.4\ \text{m}$$

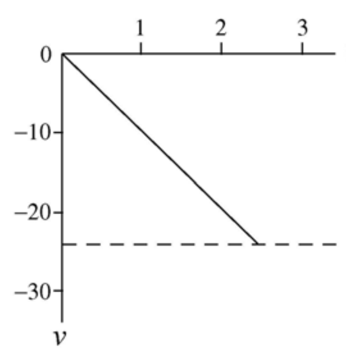
so that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24\ \text{m/s})}{9.8\ \text{m/s}^2} = 2.45\ \text{s}.$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at -9.8 m/s^2.





47. We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a = constant motion.

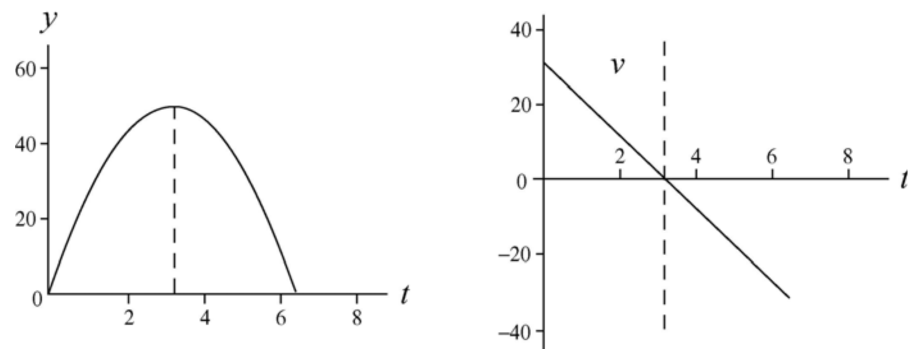
(a) At the highest point the velocity of the ball vanishes. Taking $y_0 = 0$, we set $v = 0$ in $v^2 = v_0^2 - 2gy$, and solve for the initial velocity: $v_0 = \sqrt{2gy}$. Since $y = 50$ m we find $v_0 = 31$ m/s.

(b) It will be in the air from the time it leaves the ground until the time it returns to the ground ($y = 0$). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time $t > 0$) we have

$$y = v_0 t - \frac{1}{2} g t^2 \;\Rightarrow\; t = \frac{2v_0}{g}$$

which (using our result from part (a)) produces $t = 6.4$ s. It is possible to obtain this without using part (a)'s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the x and v graphs shown. In the interest of saving space, we do not show the graph of a, which is a horizontal line at -9.8 m/s^2.



49. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We are placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m.

(a) We solve $y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time, with $y = 0$, using the quadratic formula (choosing the positive root to yield a positive value for t).

$$t = \frac{v_0 + \sqrt{v_0^2 + 2 g y_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to

$$v = v_0 - gt = 12 - (9.8)(5.4) = -41 \text{ m/s}.$$

Its final *speed* is 41 m/s.

51. The speed of the boat is constant, given by $v_b = d/t$. Here, d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling. To calculate t, we put the origin of the coordinate system at the point where the key is dropped and take the y axis to be positive in the *downward* direction. Taking the time to be zero at the instant the key is dropped, we compute the time t when $y = 45$ m. Since the initial velocity of the key is zero, the coordinate of the key is given by $y = \frac{1}{2}gt^2$. Thus

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is

$$v_b = \frac{12\text{ m}}{3.03\text{ s}} = 4.0\text{ m/s}.$$

55. (a) We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0\times10^{-3}$ s is the duration of contact. Now, to find the velocity just *before* contact, we put the origin at the point where the ball is dropped (and take $+y$ upward) and take $t = 0$ to be when it is dropped. The ball strikes the ground at $y = -15.0$ m. Its velocity there is found from Eq. 2-16: $v^2 = -2gy$. Therefore,

$$v = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-15.0)} = -17.1\text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{0-(-17.1)}{20.0\times10^{-3}} = 857\text{ m/s}^2.$$

89. Integrating (from $t = 2$ s to variable $t = 4$ s) the acceleration to get the velocity (and using the velocity datum mentioned in the problem, leads to

$$v = 17 + \tfrac{1}{2}(5)(4^2 - 2^2) = 47\text{ m/s}.$$

91. We take $+x$ in the direction of motion, so

$$v = (60\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right) = +16.7\ \text{m/s}$$

and $a > 0$. The location where it starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

(a) Eq. 2-7 gives $a_{avg} = (v - v_0)/t$ where $t = 5.4$ s and the velocities are given above. Thus, $a_{avg} = 3.1\ \text{m/s}^2$.

(b) The assumption that a = constant permits the use of Table 2-1. From that list, we choose Eq. 2-17:

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7\ \text{m/s})(5.4\ \text{s}) = 45\ \text{m}.$$

(c) We use Eq. 2-15, now with $x = 250$ m:

$$x = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\ \text{m})}{3.1\ \text{m/s}^2}}$$

which yields $t = 13$ s.

97. The (ideal) driving time before the change was $t = \Delta x/v$, and after the change it is $t' = \Delta x/v'$. The time saved by the change is therefore

$$t - t' = \Delta x\left(\frac{1}{v} - \frac{1}{v'}\right) = \Delta x\left(\frac{1}{55} - \frac{1}{65}\right) = \Delta x(0.0028\ \text{h/mi})$$

which becomes, converting $\Delta x = 700/1.61 = 435$ mi (using a conversion found on the inside front cover of the textbook), $t - t' = (435)(0.0028) = 1.2$ h. This is equivalent to 1 h and 13 min.

99. We neglect air resistance, which justifies setting $a = -g = -9.8\ \text{m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-31), the time of flight t is half of its time of ascent t_a, which is given by Eq. 2-18 with $\Delta y = H$ and $v = 0$ (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \quad\Rightarrow\quad t_a = \sqrt{\frac{2H}{g}}$$

Writing these in terms of the total time in the air $t = 2t_a$ we have

$$H = \frac{1}{8} g t^2 \quad \Rightarrow \quad t = 2\sqrt{\frac{2H}{g}}.$$

We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{\frac{1}{8} g t_2^2}{\frac{1}{8} g t_1^2} = \left(\frac{t_2}{t_1}\right)^2$$

from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2 H_1 = 4H_1$.

107. (a) The wording of the problem makes it clear that the equations of Table 2-1 apply, the challenge being that v_0, v, and a are not explicitly given. We can, however, apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ to a variety of points on the graph and solve for the unknowns from the simultaneous equations. For instance,

$$16 - 0 = v_0(2.0) + \frac{1}{2}a(2.0)^2$$

$$27 - 0 = v_0(3.0) + \frac{1}{2}a(3.0)^2$$

lead to the values $v_0 = 6.0$ m/s and $a = 2.0$ m/s^2.

(b) From Table 2-1,

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \Rightarrow \quad 27 - 0 = v(3.0) - \frac{1}{2}(2.0)(3.0)^2$$

which leads to $v = 12$ m/s.

(c) Assuming the wind continues during $3.0 \le t \le 6.0$, we apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ to this interval (where $v_0 = 12.0$ m/s from part (b)) to obtain

$$\Delta x = (12.0)(3.0) + \frac{1}{2}(2.0)(3.0)^2 = 45 \text{ m} .$$

Chapter 3 – Student Solutions Manual

1. A vector $\vec{a}$ can be represented in the *magnitude-angle* notation (a, θ), where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

is the angle $\vec{a}$ makes with the positive x axis.

(a) Given $A_x = -25.0$ m and $A_y = 40.0$ m, $A = \sqrt{(-25.0 \text{ m})^2 + (40.0 \text{ m})^2} = 47.2 \text{ m}$

(b) Recalling that tan θ = tan (θ + 180°), $\tan^{-1}$ [40/ (– 25)] = – 58° or 122°. Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility.

3. The x and the y components of a vector $\vec{a}$ lying on the xy plane are given by

$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

where $a = |\vec{a}|$ is the magnitude and θ is the angle between $\vec{a}$ and the positive x axis.

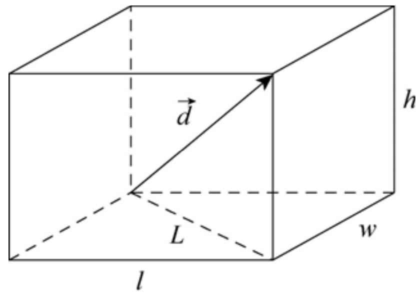
(a) The x component of $\vec{a}$ is given by a_x = 7.3 cos 250° = – 2.5 m.

(b) and the y component is given by a_y = 7.3 sin 250° = – 6.9 m.

In considering the variety of ways to compute these, we note that the vector is 70° below the – x axis, so the components could also have been found from a_x = – 7.3 cos 70° and a_y = – 7.3 sin 70°. In a similar vein, we note that the vector is 20° to the left from the – y axis, so one could use a_x = – 7.3 sin 20° and a_y = – 7.3 cos 20° to achieve the same results.

7. The length unit meter is understood throughout the calculation.

(a) We compute the distance from one corner to the diametrically opposite corner:
$d = \sqrt{3.00^2 + 3.70^2 + 4.30^2} = 6.42$.

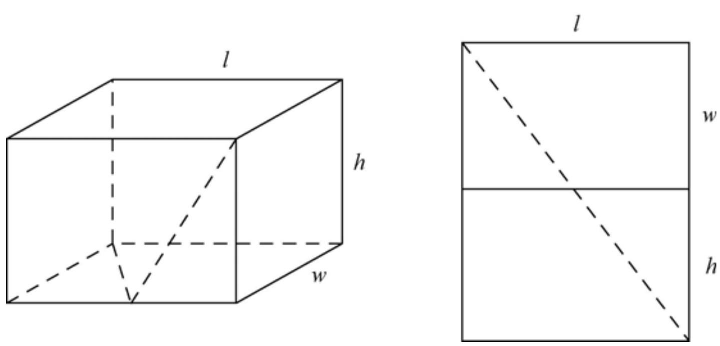


(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the *x* axis to be out of the page, the *y* axis to be to the right, and the *z* axis to be upward. Then the *x* component of the displacement is *w* = 3.70, the *y* component of the displacement is 4.30, and the *z* component is 3.00. Thus $\vec{d} = 3.70\,\hat{\mathrm{i}} + 4.30\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$. An equally correct answer is gotten by interchanging the length, width, and height.



(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\min} = \sqrt{(w+h)^2 + \ell^2} = \sqrt{(3.70 + 3.00)^2 + 4.30^2} = 7.96\,\mathrm{m}\,.$$

9. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters.

(a) The x and the y components of $\vec{r}$ are $r_x = a_x + b_x$ = (4.0 m) – (13 m) = –9.0 m and $r_y = a_y + b_y$ = (3.0 m) + (7.0 m) = 10 m, respectively. Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\text{i}} + (10\,\text{m})\,\hat{\text{j}}$.

(b) The magnitude of $\vec{r}$ is

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}.$$

(c) The angle between the resultant and the $+x$ axis is given by

$$\theta = \tan^{-1}(r_y/r_x) = \tan^{-1}[(10\text{ m})/(-9.0\text{ m})] = -48° \text{ or } 132°.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

17. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the $+x$ axis is 30° +105° = 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The x component of $\vec{r}$ is r_x = (10.0 m) cos 30° + (10.0 m) cos 135° = 1.59 m.

(b) The y component of $\vec{r}$ is r_y = (10.0 m) sin 30° + (10.0 m) sin 135° = 12.1 m.

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59\text{ m})^2 + (12.1\text{ m})^2} = 12.2\text{ m}$.

(d) The angle between $\vec{r}$ and the $+x$ direction is $\tan^{-1}$[(12.1 m)/(1.59 m)] = 82.5°.

39. Since $ab\cos\phi = a_xb_x + a_yb_y + a_zb_z$,

$$\cos\phi = \frac{a_xb_x + a_yb_y + a_zb_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a = |\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

$$b = |\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74.$$

The angle between them is found from

$$\cos\phi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22°$.

43. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is 120°. Direct application of Eq. 3-5 yields the answers for this and the next few parts.

(a) $a_x = a \cos 0° = a = 3.00$ m.

(b) $a_y = a \sin 0° = 0$.

(c) $b_x = b \cos 30° = (4.00 \text{ m}) \cos 30° = 3.46$ m.

(d) $b_y = b \sin 30° = (4.00 \text{ m}) \sin 30° = 2.00$ m.

(e) $c_x = c \cos 120° = (10.0 \text{ m}) \cos 120° = –5.00$ m.

(f) $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) In terms of components (first x and then y), we must have

$$\begin{aligned} -5.00 \text{ m} &= p\,(3.00 \text{ m}) + q\,(3.46 \text{ m}) \\ 8.66 \text{ m} &= p\,(0) + q\,(2.00 \text{ m}). \end{aligned}$$

Solving these equations, we find $p = –6.67$.

(h) And $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

47. We apply Eq. 3-20 and Eq. 3-27.

(a) The scalar (dot) product of the two vectors is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = (10)\,(6.0)\cos 60° = 30.$$

(b) The magnitude of the vector (cross) product of the two vectors is

$$|\,\vec{a}\times\vec{b}\,| = ab\sin\phi = (10)\,(6.0)\sin 60° = 52.$$

51. Let $\vec{A}$ represent the first part of his actual voyage (50.0 km east) and $\vec{C}$ represent the intended voyage (90.0 km north). We are looking for a vector $\vec{B}$ such that $\vec{A} + \vec{B} = \vec{C}$.

(a) The Pythagorean theorem yields $B = \sqrt{(50.0)^2 + (90.0)^2} = 103\ \text{km}$.

(b) The direction is $\tan^{-1}(50.0/90.0) = 29.1°$ west of north (which is equivalent to 60.9° north of due west).

71. Given: $\vec{A} + \vec{B} = 6.0\,\hat{i} + 1.0\,\hat{j}$ and $\vec{A} - \vec{B} = -4.0\,\hat{i} + 7.0\,\hat{j}$. Solving these simultaneously leads to $\vec{A} = 1.0\,\hat{i} + 4.0\,\hat{j}$. The Pythagorean theorem then leads to $A = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$.

11. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\mathrm{i}} + 4t^2\,\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}) = 8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}\,.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}\,(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\,.$$

17. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{\mathrm{i}}\right)\left(3.00\right) + \frac{1}{2}\left(-1.00\,\hat{\mathrm{i}} - 0.50\,\hat{\mathrm{j}}\right)\left(3.00\right)^2 = (4.50\,\hat{\mathrm{i}} - 2.25\,\hat{\mathrm{j}}) \text{ m}.$$

29. The initial velocity has no vertical component — only an x component equal to +2.00 m/s. Also, $y_0 = +10.0$ m if the water surface is established as $y = 0$.

(a) $x - x_0 = v_x t$ readily yields $x - x_0 = 1.60$ m.

(b) Using $y - y_0 = v_{0y} t - \frac{1}{2}gt^2$, we obtain $y = 6.86$ m when $t = 0.800$ s and v_{0y}=0.

(c) Using the fact that $y = 0$ and $y_0 = 10.0$, the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ leads to $t = \sqrt{2(10.0\text{ m})/9.80\text{ m/s}^2} = 1.43\text{ s}$. During this time, the x-displacement of the diver is $x - x_0 = (2.00\text{ m/s})(1.43\text{ s}) = 2.86\text{ m}$.

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0°$ for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \quad\Rightarrow\quad 0 - 730\text{ m} = v_0 \sin(-37.0°)(5.00\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(5.00\text{ s})^2$$

which yields $v_0 = 202$ m/s.

(b) The horizontal distance traveled is $x = v_0 t\cos\theta_0 = (202\text{ m/s})(5.00\text{ s})\cos(-37.0°) = 806\text{ m}$.

(c) The x component of the velocity (just before impact) is

$$v_x = v_0\cos\theta_0 = (202\text{ m/s})\cos(-37.0°) = 161\text{ m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202\text{ m/s})\sin(-37.0°) - (9.80\text{ m/s}^2)(5.00\text{ s}) = -171\text{ m/s}.$$

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t\cos\theta_0$ and $0 = v_0 t\sin\theta_0 - \frac{1}{2}gt^2$. Eliminating t leads to $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$. Using $\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\,(2\theta_0) = gd \;\Rightarrow\; \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\text{ m/s}^2)(45.7\text{ m})}{(460\text{ m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\text{ m})\tan(0.0606°) = 0.0484\text{ m} = 4.84\text{ cm}.$$

47. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at x = 97.5 m, which requires knowing the initial velocity. Using the range information and θ_0 = 45°, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{\left(9.8\text{ m/s}^2\right)\left(107\text{ m}\right)}{1}} = 32.4\text{ m/s}.$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{\left(32.4\text{ m/s}\right)\cos 45°} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + \left(v_0\sin\theta_0\right)t - \frac{1}{2}gt^2 = 9.88\text{ m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At t = 4.26 s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

51. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0\cos\theta_0,\quad y = v_0 t\sin\theta_0 - \tfrac{1}{2}gt^2,$$

we see the first equation gives $t = x/v_0\cos\theta_0$, and when this is substituted into the second the result is

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0}.$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity $1/\cos^2\theta_0 = 1 + \tan^2\theta_0$, we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for tan θ_0. To simplify writing the solution, we denote $c = \frac{1}{2}gx^2/v_0^2 = \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(50\text{ m}\right)^2/\left(25\text{ m/s}\right)^2 = 19.6\text{m}.$ Then the second-order equation becomes $c\tan^2\theta_0 - x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50\text{ m} \pm \sqrt{(50\text{ m})^2 - 4(3.44\text{ m} + 19.6\text{ m})(19.6\text{ m})}}{2(19.6\text{ m})}.$$

The two solutions are given by tan θ_0 = 1.95 and tan θ_0 = 0.605. The corresponding (first-quadrant) angles are θ_0 = 63° and θ_0 = 31°. Thus,

(a) The smallest elevation angle is θ_0 = 31°, and

(b) The greatest elevation angle is θ_0 = 63°.

If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

53. We denote *h* as the height of a step and *w* as the width. To hit step *n*, the ball must fall a distance *nh* and travel horizontally a distance between (*n* – 1)*w* and *nw*. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the *y* axis to be positive in the upward direction. The coordinates of the ball at time *t* are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since v_{0y} = 0). We equate *y* to –*nh* and solve for the time to reach the level of step *n*:

$$t = \sqrt{\frac{2nh}{g}}.$$

The *x* coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52\text{ m/s})\sqrt{\frac{2n(0.203\text{ m})}{9.8\text{ m/s}^2}} = (0.309\text{ m})\sqrt{n}.$$

The method is to try values of *n* until we find one for which *x*/*w* is less than *n* but greater than *n* – 1. For *n* = 1, *x* = 0.309 m and *x*/*w* = 1.52, which is greater than *n*. For *n* = 2, *x* = 0.437 m and *x*/*w* = 2.15, which is also greater than *n*. For *n* = 3, *x* = 0.535 m and *x*/*w* = 2.64. Now, this is less than *n* and greater than *n* – 1, so the ball hits the third step.

67. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

75. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9\text{ m/s}}{8.0\text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

77. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is $v_h = 30$ m/s, the same as the speed of the train. If v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then $\tan\theta = v_h/v_v$. Thus $v_v = v_h/\tan\theta = (30\text{ m/s})/\tan 70° = 10.9$ m/s. The speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30\text{ m/s})^2 + (10.9\text{ m/s})^2} = 32\text{ m/s}.$$

91. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the firing point, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t = \sqrt{\frac{2(0.019 \text{ m})}{9.8 \text{ m/s}^2}} = 6.2\times 10^{-2} \text{ s}.$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since x = 30 m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{30 \text{ m}}{6.3\times 10^{-2} \text{ s}} = 4.8\times 10^2 \text{ m/s}.$$

107. (a) Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when t = 3.0 s and v_y = 0):

$$y_{\max} - y_0 = v_y t - \frac{1}{2}gt^2 \ .$$

With ground level chosen so y_0 = 0, this equation gives the result $y_{\max} = \frac{1}{2}g(3.0 \text{ s})^2$ = 44 m.

(b) After the moment it reached maximum height, it is falling; at t = 2.5 s, it will have fallen an amount given by Eq. 2-18

$$y_{\text{fence}} - y_{\max} = (0)(2.5 \text{ s}) - \frac{1}{2}\, g(2.5 \text{ s})^2$$

which leads to y_{fence} = 13 m.

(c) Either the *range* formula, Eq. 4-26, can be used or one can note that after passing the fence, it will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find the original x-component from 97.5 m = v_{0x}(5.5 s) and then apply it to that final 0.5 s. Thus, we find v_{0x} = 17.7 m/s and that after the fence

$$\Delta x = (17.7 \text{ m/s})(0.5 \text{ s}) = 8.9 \text{ m}.$$

111. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15)

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\,\hat{\text{j}}\right)t + \frac{1}{2}\left(4.0\,\hat{\text{i}} + 2.0\,\hat{\text{j}}\right)t^2 = \left(2.0t^2\right)\hat{\text{i}} + \left(8.0t + 1.0t^2\right)\hat{\text{j}}.$$

Therefore, we find when x = 29 m, by solving $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is y = (8.0 m/s)(3.8 s) + (1.0 m/s^2)(3.8 s)^2 = 45 m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t.$$

Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0 \text{ m/s})\,\hat{\text{j}} + \left((4.0 \text{ m/s}^2)\,\hat{\text{i}} + (2.0 \text{ m/s}^2)\,\hat{\text{j}}\right)(3.8 \text{ s}) = (15.2 \text{ m/s})\,\hat{\text{i}} + (15.6 \text{ m/s})\,\hat{\text{j}}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2 \text{ m/s})^2 + (15.6 \text{ m/s})^2} = 22 \text{ m/s}.$$

121. On the one hand, we could perform the vector addition of the displacements with a vector-capable calculator in polar mode $((75 \angle 37°) + (65 \angle -90°) = (63 \angle -18°))$, but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with $+x$ East and $+y$ North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

$$\begin{aligned} \Delta\vec{r}_1 &= (75 \text{ km})\cos(37°)\,\hat{\text{i}} + (75 \text{ km})\sin(37°)\,\hat{\text{j}} \\ \Delta\vec{r}_2 &= (-65 \text{ km})\,\hat{\text{j}} \\ \Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 &= (60 \text{ km})\,\hat{\text{i}} - (20 \text{ km})\,\hat{\text{j}}\,. \end{aligned}$$

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length $|\Delta\vec{r}| = \sqrt{(60 \text{ km})^2 + (-20 \text{ km})^2} = 63 \text{ km}$.

(b) The direction of $\Delta\vec{r}$ is $\theta = \tan^{-1}[(-20 \text{ km})/(60 \text{ km})] = -18°$, or $18°$ south of east.

(c) We use the result from part (a) in Eq. 4-8 along with the fact that $\Delta t = 90$ h. In unit vector notation, we obtain

$$\vec{v}_{\text{avg}} = \frac{(60\,\hat{\text{i}} - 20\,\hat{\text{j}}) \text{ km}}{90 \text{ h}} = (0.67\,\hat{\text{i}} - 0.22\,\hat{\text{j}}) \text{ km/h}.$$

This leads to $|\vec{v}_{\text{avg}}| = 0.70 \text{ km/h}$.

(d) The direction of $\vec{v}_{avg}$ is $\theta = \tan^{-1}[(-0.22 \text{ km/h})/(0.67 \text{ km/h})] = -18°$, or 18° south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h ≈ 1.6 km/h .

(f) The net displacement is required to be the 90 km East from *A* to *B*. The displacement from the resting place to *B* is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90 \text{ km})\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30 \text{ km})\hat{\text{i}} + (20 \text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36 \angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\text{avg}}| = \frac{36 \text{ km}}{(120 - 90) \text{ h}} = 1.2 \text{ km/h}.$$

(g) The direction of $\vec{v}_{avg}$ is the same as $\vec{r}_3$ (that is, 33° north of east).

5. We denote the two forces $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

(a) In unit vector notation $\vec{F}_1 = (20.0\ \text{N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0 \sin 30.0°\,\text{m/s}^2\right)\hat{\text{i}} - \left(12.0 \cos 30.0°\,\text{m/s}^2\right)\hat{\text{j}} = -\left(6.00\ \text{m/s}^2\right)\hat{\text{i}} - \left(10.4\text{m/s}^2\right)\hat{\text{j}}.$$

Therefore,

$$\begin{aligned}\vec{F}_2 &= (2.00\,\text{kg})\left(-6.00\,\text{m/s}^2\right)\hat{\text{i}} + (2.00\ \text{kg})\left(-10.4\ \text{m/s}^2\right)\hat{\text{j}} - (20.0\ \text{N})\hat{\text{i}} \\ &= (-32.0\ \text{N})\hat{\text{i}} - (20.8\ \text{N})\hat{\text{j}}.\end{aligned}$$

(b) The magnitude of $\vec{F}_2$ is

$$|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0)^2 + (-20.8)^2} = 38.2\ \text{N}.$$

(c) The angle that $\vec{F}_2$ makes with the positive x axis is found from

$$\tan\theta = (F_{2y}/F_{2x}) = [(-20.8)/(-32.0)] = 0.656.$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is 213°. An alternative answer is $213° - 360° = -147°$.

13. (a) – (c) In all three cases the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami. Its value is (11.0 kg) (9.8 m/s^2) = 108 N.

19. (a) Since the acceleration of the block is zero, the components of the Newton's second law equation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0.$$

Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\ \text{kg})\left(9.8\ \text{m/s}^2\right)\sin 30° = 42\ \text{N}.$$

(b) We solve the second equation in part (a) for the normal force F_N:

$$F_N = mg\cos\theta = (8.5\ \text{kg})(9.8\ \text{m/s}^2)\ \cos\ 30° = 72\ \text{N}\ .$$

(c) When the string is cut, it no longer exerts a force on the block and the block accelerates. The x component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

$$a = -g\sin\ \theta = -9.8\sin 30° = -4.9\ \text{m/s}^2.$$

The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

25. In terms of magnitudes, Newton's second law is $F = ma$, where $F = \left|\vec{F}_{\text{net}}\right|,\ a = |\vec{a}\,|$, and m is the (always positive) mass. The magnitude of the acceleration can be found using constant acceleration kinematics (Table 2-1). Solving $v = v_0 + at$ for the case where it starts from rest, we have $a = v/t$ (which we interpret in terms of magnitudes, making specification of coordinate directions unnecessary). The velocity is v = (1600 km/h) (1000 m/km)/(3600 s/h) = 444 m/s, so

$$F = (500\,\text{kg})\ \frac{444\ \text{m/s}}{1.8\,\text{s}} = 1.2 \times 10^5\ \text{N}.$$

29. The acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. The force of gravity is negligible. We take the $+x$ axis to be in the direction of the initial velocity and the $+y$ axis to be in the direction of the electrical force, and place the origin at the initial position of the electron. Since the force and acceleration are constant, we use the equations from Table 2-1: $x = v_0t$ and

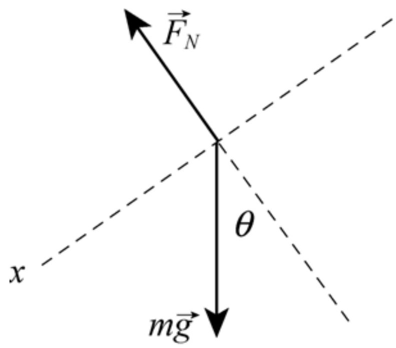
$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2\ .$$

The time taken by the electron to travel a distance x (= 30 mm) horizontally is $t = x/v_0$ and its deflection in the direction of the force is

$$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5\times10^{-16}}{9.11\times10^{-31}}\right)\left(\frac{30\times10^{-3}}{1.2\times10^7}\right)^2 = 1.5\times10^{-3}\ \text{m}\ .$$

35. The free-body diagram is shown next. $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. We take the $+x$ direction to be down the incline, in the direction of the acceleration, and the $+y$ direction to be in the direction

of the normal force exerted by the incline on the block. The x component of Newton's second law is then $mg \sin \theta = ma$; thus, the acceleration is $a = g \sin \theta$.



(a) Placing the origin at the bottom of the plane, the kinematic equations (Table 2-1) for motion along the x axis which we will use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. The block momentarily stops at its highest point, where $v = 0$; according to the second equation, this occurs at time $t = -v_0/a$. The position where it stops is

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{(-3.50\ \text{m/s})^2}{\left(9.8\ \text{m/s}^2\right)\sin\ 32.0°}\right) = -1.18\ \text{m},$$

or $|x| = 1.18$ m.

(b) The time is

$$t = \frac{v_0}{a} = -\frac{v_0}{g \sin\ \theta} = -\frac{-3.50\,\text{m/s}}{\left(9.8\,\text{m/s}^2\right)\sin\ 32.0°} = 0.674\,\text{s}.$$

(c) That the return-speed is identical to the initial speed is to be expected since there are no dissipative forces in this problem. In order to prove this, one approach is to set $x = 0$ and solve $x = v_0 t + \frac{1}{2}at^2$ for the total time (up and back down) t. The result is

$$t = -\frac{2v_0}{a} = -\frac{2v_0}{g \sin\theta} = -\frac{2(-3.50\ \text{m/s})}{\left(9.8\ \text{m/s}^2\right)\sin\ 32.0°} = 1.35\ \text{s}.$$

The velocity when it returns is therefore

$$v = v_0 + at = v_0 + gt\ \sin\ \theta = -3.50 + (9.8)\,(1.35)\ \sin\ 32° = 3.50\ \text{m/s}.$$

45. (a) The links are numbered from bottom to top. The forces on the bottom link are the force of gravity $m\vec{g}$, downward, and the force $\vec{F}_{2\text{on}1}$ of link 2, upward. Take the positive direction to be upward. Then Newton's second law for this link is $F_{2\text{on}1} - mg = ma$. Thus,

$$F_{2\text{on}1} = m(a + g) = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 1.23 \text{ N}.$$

(b) The forces on the second link are the force of gravity $m\vec{g}$, downward, the force $\vec{F}_{1\text{on}2}$ of link 1, downward, and the force $\vec{F}_{3\text{on}2}$ of link 3, upward. According to Newton's third law $\vec{F}_{1\text{on}2}$ has the same magnitude as $\vec{F}_{2\text{on}1}$. Newton's second law for the second link is $F_{3\text{on}2} - F_{1\text{on}2} - mg = ma$, so

$$F_{3\text{on}2} = m(a + g) + F_{1\text{on}2} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 1.23 \text{ N} = 2.46 \text{ N}.$$

(c) Newton's second for link 3 is $F_{4\text{on}3} - F_{2\text{on}3} - mg = ma$, so

$$F_{4\text{on}3} = m(a + g) + F_{2\text{on}3} = (0.100 \text{ N})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 2.46 \text{ N} = 3.69 \text{ N},$$

where Newton's third law implies $F_{2\text{on}3} = F_{3\text{on}2}$ (since these are magnitudes of the force vectors).

(d) Newton's second law for link 4 is $F_{5\text{on}4} - F_{3\text{on}4} - mg = ma$, so

$$F_{5\text{on}4} = m(a + g) + F_{3\text{on}4} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 3.69 \text{ N} = 4.92 \text{ N},$$

where Newton's third law implies $F_{3\text{on}4} = F_{4\text{on}3}$.

(e) Newton's second law for the top link is $F - F_{4\text{on}5} - mg = ma$, so

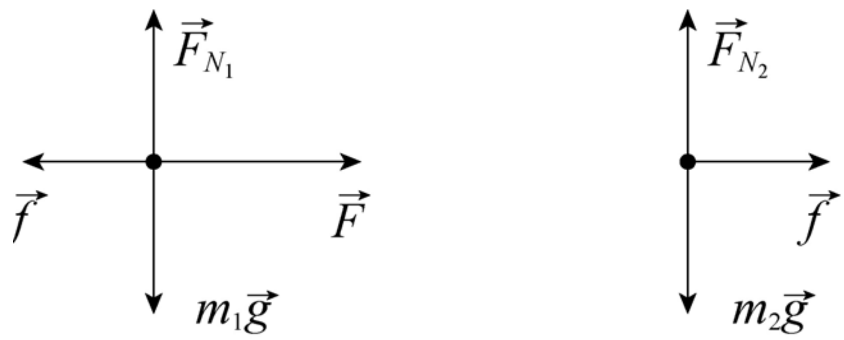
$$F = m(a + g) + F_{4\text{on}5} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4.92 \text{ N} = 6.15 \text{ N},$$

where $F_{4\text{on}5} = F_{5\text{on}4}$ by Newton's third law.

(f) Each link has the same mass and the same acceleration, so the same net force acts on each of them:

$$F_{\text{net}} = ma = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2) = 0.250 \text{ N}.$$

53. The free-body diagrams for part (a) are shown below. $\vec{F}$ is the applied force and $\vec{f}$ is the force exerted by block 1 on block 2. We note that $\vec{F}$ is applied directly to block 1 and that block 2 exerts the force $-\vec{f}$ on block 1 (taking Newton's third law into account).



(a) Newton's second law for block 1 is $F - f = m_1a$, where a is the acceleration. The second law for block 2 is $f = m_2a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations. From the second equation we obtain the expression $a = f/m_2$, which we substitute into the first equation to get $F - f = m_1f/m_2$. Therefore,

$$f = \frac{Fm_2}{m_1 + m_2} = \frac{(3.2\ \text{N})\,(1.2\ \text{kg})}{2.3\ \text{kg} + 1.2\ \text{kg}} = 1.1\ \text{N}\ .$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1 (and in the opposite direction), the force of contact between the blocks is

$$f = \frac{Fm_1}{m_1 + m_2} = \frac{(3.2\ \text{N})\,(2.3\ \text{kg})}{2.3\ \text{kg} + 1.2\ \text{kg}} = 2.1\ \text{N}\ .$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force f is the only horizontal force on the block of mass m_2 and in part (b) f is the only horizontal force on the block with $m_1 > m_2$. Since $f = m_2a$ in part (a) and $f = m_1a$ in part (b), then for the accelerations to be the same, f must be larger in part (b).

57. We take $+y$ to be up for both the monkey and the package.

(a) The force the monkey pulls downward on the rope has magnitude F. According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to

$$F - m_mg = m_ma_m,$$

where m_m is the mass of the monkey and a_m is its acceleration. Since the rope is massless $F = T$ is the tension in the rope. The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is

$$F + F_N - m_pg = m_pa_p,$$

where m_p is the mass of the package, a_p is its acceleration, and F_N is the normal force exerted by the ground on it. Now, if F is the minimum force required to lift the package, then $F_N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$. Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{10\text{ kg}} = 4.9\text{ m/s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a_p$ for the package and $F - m_m g = m_m a_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a_m = -a_p$. Solving the first equation for F

$$F = m_p(g + a_p) = m_p(g - a_m)$$

and substituting this result into the second equation, we solve for a_m:

$$a_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m/s}^2.$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, we obtain

$$F = m_p(g - a_m) = (15\text{ kg})(9.8\text{ m/s}^2 - 2.0\text{ m/s}^2) = 120\text{ N}.$$

61. The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air $\vec{F}_a$ (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration (which is not its usual use in this chapter). When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is

$$F_a - Mg = -Ma.$$

After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is upward. Newton's second law leads to

$$F_a - (M - m)g = (M - m)a.$$

The previous equation gives $F_a = M(g - a)$, and this plugs into the new equation to give

$$M(g - a) - (M - m)g = (M - m)a \;\Rightarrow\; m = \frac{2Ma}{g + a}.$$

73. Although the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y axes, only the x-application is needed to find what this particular problem asks for. We note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. We choose $+x$ to the right and $+y$ up. We also note that the x component of the rope's tension (acting on the crate) is

$$F_x = F\cos\theta = 450 \cos 38° = 355 \text{ N},$$

and the resistive force (pointing in the $-x$ direction) has magnitude $f = 125$ N.

(a) Newton's second law leads to

$$F_x - f = ma \Rightarrow a = \frac{355 \text{ N} - 125 \text{ N}}{310 \text{ kg}} = 0.74 \text{m/s}^2 .$$

(b) In this case, we use Eq. 5-12 to find the mass: $m = W/g = 31.6$ kg. Now, Newton's second law leads to

$$T_x - f = ma \Rightarrow a = \frac{355 \text{ N} - 125 \text{ N}}{31.6 \text{ kg}} = 7.3 \text{ m/s}^2 .$$

79. The "certain force" denoted F is assumed to be the net force on the object when it gives m_1 an acceleration $a_1 = 12$ m/s^2 and when it gives m_2 an acceleration $a_2 = 3.3$ m/s^2. Thus, we substitute $m_1 = F/a_1$ and $m_2 = F/a_2$ in appropriate places during the following manipulations.

(a) Now we seek the acceleration a of an object of mass $m_2 - m_1$ when F is the net force on it. Thus,

$$a = \frac{F}{m_2 - m_1} = \frac{F}{(F/a_2) - (F/a_1)} = \frac{a_1 a_2}{a_1 - a_2}$$

which yields $a = 4.6$ m/s^2.

(b) Similarly for an object of mass $m_2 + m_1$:

$$a = \frac{F}{m_2 + m_1} = \frac{F}{(F/a_2) + (F/a_1)} = \frac{a_1 a_2}{a_1 + a_2}$$

which yields $a = 2.6$ m/s^2.

91. (a) The bottom cord is only supporting $m_2 = 4.5$ kg against gravity, so its tension is $T_2 = m_2 g = (4.5 \text{ kg})(9.8 \text{ m/s}^2) = 44$ N.

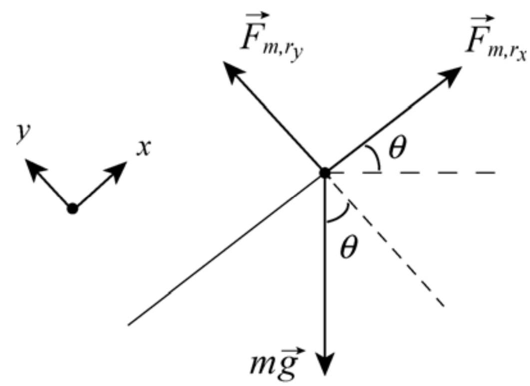
(b) The top cord is supporting a total mass of $m_1 + m_2 = (3.5\text{ kg} + 4.5\text{ kg}) = 8.0$ kg against gravity, so the tension there is

$$T_1 = (m_1 + m_2)g = (8.0\text{ kg})(9.8\text{ m/s}^2) = 78\text{ N}.$$

(c) In the second picture, the lowest cord supports a mass of $m_5 = 5.5$ kg against gravity and consequently has a tension of $T_5 = (5.5\text{ kg})(9.8\text{ m/s}^2) = 54$ N.

(d) The top cord, we are told, has tension T_3 =199 N which supports a total of (199 N)/(9.80 m/s^2) = 20.3 kg, 10.3 kg of which is already accounted for in the figure. Thus, the unknown mass in the middle must be m_4 = 20.3 kg – 10.3 kg = 10.0 kg, and the tension in the cord above it must be enough to support $m_4 + m_5$ = (10.0 kg + 5.50 kg) = 15.5 kg, so T_4 = (15.5 kg)(9.80 m/s^2) = 152 N. Another way to analyze this is to examine the forces on m_3; one of the downward forces on it is T_4.

95. The free-body diagrams is shown on the right. Note that $F_{\text{m, r}_y}$ and $F_{\text{m, r}_x}$, respectively, and thought of as the y and x components of the force $\vec{F}_{\text{m, r}}$ exerted by the motorcycle on the rider.



(a) Since the net force equals ma, then the magnitude of the net force on the rider is (60.0 kg) (3.0 m/s^2) = 1.8 × 10^2 N.

(b) We apply Newton's second law to the x axis:

$$F_{\text{m,r}_x} - mg\sin\theta = ma$$

where m = 60.0 kg, a = 3.0 m/s^2, and θ = 10°. Thus, $F_{\text{m, r}_x} = 282$ N Applying it to the y axis (where there is no acceleration), we have

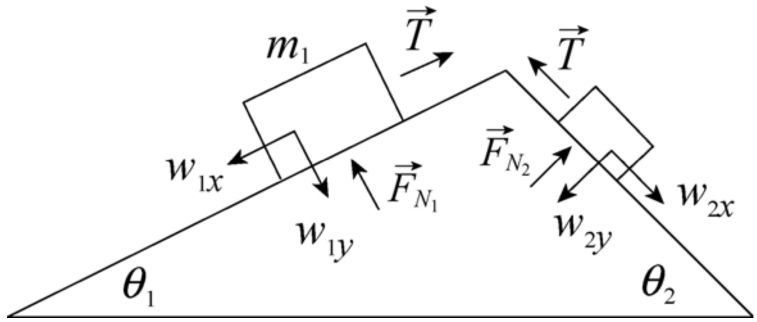
$$F_{\text{m,r}_y} - mg\cos\theta = 0$$

which produces $F_{\text{m, r}_y}$ = 579 N . Using the Pythagorean theorem, we find

$$\sqrt{F_{m,r_x}^2 + F_{m,r_y}^2} = 644\text{ N}.$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 6.4 × 10^2 N.

99. The +x axis is "uphill" for m_1 = 3.0 kg and "downhill" for m_2 = 2.0 kg (so they both accelerate with the same sign). The x components of the two masses along the x axis are given by $w_{1x} = m_1 g\sin\theta_1$ and $w_{2x} = m_2 g\sin\theta_2$, respectively.



Applying Newton's second law, we obtain

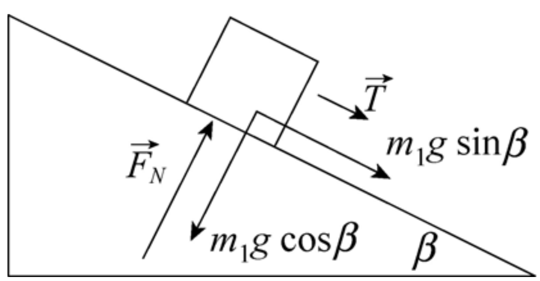
$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a$$

Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1}\right) g$$

With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have a = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension T = 16 N.

101. We first analyze the forces on m_1=1.0 kg.



The +x direction is "downhill" (parallel to $\vec{T}$).

With the acceleration (5.5 m/s^2) in the positive x direction for m_1, then Newton's second law, applied to the x axis, becomes

$$T + m_1 g \sin\beta = m_1 \left(5.5\,\text{m/s}^2\right)$$

But for m_2=2.0 kg, using the more familiar vertical y axis (with *up* as the positive direction), we have the acceleration in the negative direction:

$$F + T - m_2 g = m_2\left(-5.5\,\text{m/s}^2\right)$$

where the tension comes in as an upward force (the cord can pull, not push).

(a) From the equation for m_2, with F = 6.0 N, we find the tension T = 2.6 N.

(b) From the equation for m, using the result from part (a), we obtain the angle $\beta = 17°$.

1. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain

$$F - f_{s,\,\max} = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\max} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

(a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to $F = 198$ N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing $m = 45$ kg with $m = 28$ kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

3. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by $f_k = \mu_k F_N$ (Eq. 6-2). Applying Newton's second law to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

(a) The second equation yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k mg = (0.35)(55\text{ kg})(9.8\text{ m/s}^2) = 1.9 \times 10^2\ \text{N}.$$

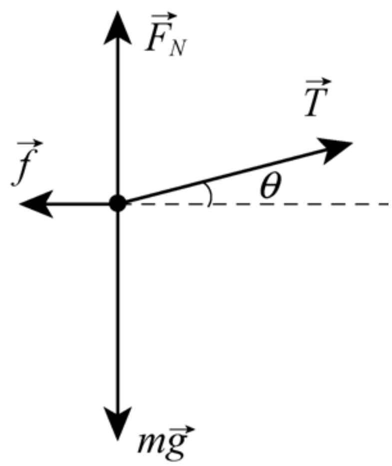
(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with F = 220 N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56 \text{ m/s}^2 .$$

13. (a) The free-body diagram for the crate is shown on the right. $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless. The equations for the x and the y components of the force according to Newton's second law are:



$$T\cos\theta - f = 0$$
$$T\sin\theta + F_N - mg = 0$$

where θ = 15° is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s (mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have

$$T\cos\theta = \mu_s (mg - T\sin\theta).$$

We solve for the tension:

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.50)\,(68\text{ kg})\,(9.8\text{ m/s}^2)}{\cos 15° + 0.50\sin 15°} = 304\text{ N} \approx 3.0\times10^2\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta - f = ma$$
$$F_N + T\sin\theta - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - T\sin\theta$, which yields $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k (mg - T\sin\theta) = ma,$$

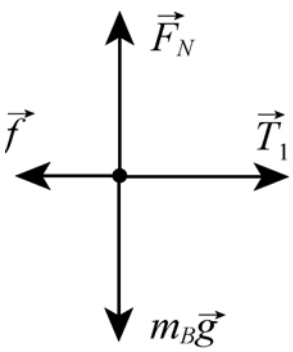
so the acceleration is

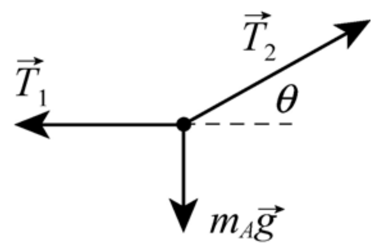
$$a = \frac{T(\cos\theta + \mu_k \sin\theta)}{m} - \mu_k g\,.$$

Numerically, it is given by

$$a = \frac{(304\ \text{N})(\cos 15° + 0.35\sin 15°)}{68\ \text{kg}} - (0.35)(9.8\ \text{m/s}^2) = 1.3\ \text{m/s}^2\,.$$

23. The free-body diagrams for block B and for the knot just above block A are shown next. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B (W_B = 711 N is the magnitude of $m_B\vec{g}$).





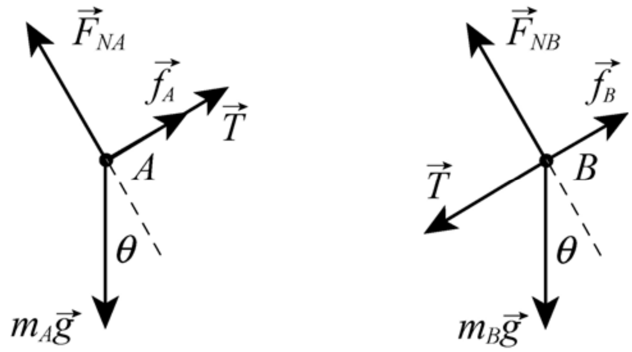
For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2\cos\theta - T_1 &= 0 \\ T_2\sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with μ_s = 0.25, we obtain $W_A = 103\ \text{N} \approx 1.0\times10^2\ \text{N}$.

27. The free-body diagrams for the two blocks are shown next. T is the magnitude of the tension force of the string, $\vec{F}_{NA}$ is the normal force on block A (the leading block), $\vec{F}_{NB}$ is the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and W_A = 3.6 N), and

m_B is the mass of block B (where $m_B = W_B/g$ and W_B = 7.2 N). The angle of the incline is θ = 30°.



For each block we take +x downhill (which is toward the lower-left in these diagrams) and +y in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B, we arrive at four equations:

$$\begin{gathered} W_A \sin\theta - f_A - T = m_A\, a \\ F_{NA} - W_A \cos\theta = 0 \\ W_B \sin\theta - f_B + T = m_B\, a \\ F_{NB} - W_B \cos\theta = 0 \end{gathered}$$

which, when combined with Eq. 6-2 ($f_A = \mu_{kA} F_{NA}$ where $\mu_{k\,A}$ = 0.10 and $f_B = \mu_{kB} F_{NB}$ f_B where $\mu_{k\,B}$ = 0.20), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

(a) From these equations, we find the acceleration to be

$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \text{m/s}^2.$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)\ (\mu_{k\,B} - \mu_{k\,A})\cos\theta = 0.21\ \text{N}.$$

Simply returning the value for a found in part (a) into one of the above equations is certainly fine, and probably easier than solving for T algebraically as we have done, but the algebraic form does illustrate the $\mu_{k\,B} - \mu_{k\,A}$ factor which aids in the understanding of the next part.

35. We denote the magnitude of the frictional force αv, where $\alpha = 70\ \text{N}\cdot\text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt}.$$

Thus,

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

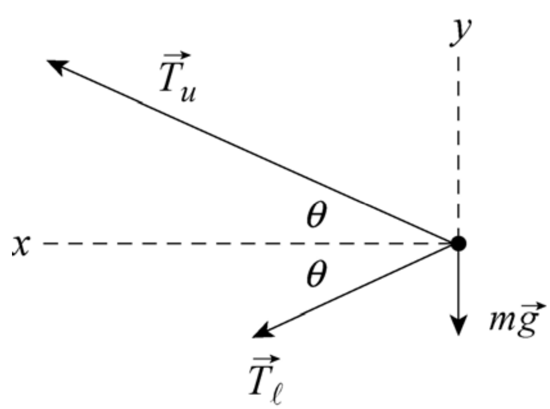
where v_0 is the velocity at time zero and v is the velocity at time t. The integrals are evaluated with the result

$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

We take $v = v_0/2$ and solve for time:

$$t = \frac{m}{\alpha}\ln 2 = \frac{1000 \text{ kg}}{70 \text{ N}\cdot\text{s/m}}\ln 2 = 9.9 \text{ s} .$$

59. The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension force of the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



(a) We take the +x direction to be leftward (toward the center of the circular orbit) and +y upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R},$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$. Since the triangle is equilateral θ = 30.0°. Thus

$$T_\ell = 35.0\ \text{N} - \frac{(1.34\ \text{kg})(9.80\ \text{m/s}^2)}{\sin 30.0°} = 8.74\ \text{N}.$$

(b) The net force has magnitude

$$F_{\text{net,str}} = \left(T_u + T_\ell\right)\cos\theta = (35.0\ \text{N} + 8.74\ \text{N})\cos 30.0° = 37.9\ \text{N}.$$

(c) The radius of the path is

$$R = ((1.70\ \text{m})/2)\tan 30.0° = 1.47\ \text{m}.$$

Using $F_{\text{net,str}} = mv^2/R$, we find that the speed of the ball is

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\ \text{m})(37.9\ \text{N})}{1.34\ \text{kg}}} = 6.45\ \text{m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward ("radially inward'').

65. (a) Using $F = \mu_s mg$, the coefficient of static friction for the surface between the two blocks is μ_s = (12 N)/(39.2 N) = 0.31, where $m_t g$ = (4.0 kg)(9.8 m/s^2)=39.2 N is the weight of the top block. Let $M = m_t + m_b = 9.0$ kg be the total *system* mass, then the maximum horizontal force has a magnitude $Ma = M\mu_s g$ = 27 N.

(b) The acceleration (in the maximal case) is $a = \mu_s g$ =3.0 m/s^2.

77. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.

(a) The horizontal component of Newton's second law is $f = mv^2/R$, where f is the static friction exerted horizontally by the ground on the tires. Thus,

$$f = \frac{(85.0\ \text{kg})(9.00\ \text{m/s})^2}{25.0\ \text{m}} = 275\ \text{N}.$$

(b) If F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$ = 833 N. The magnitude of the force exerted by the ground on the bicycle is therefore

$$\sqrt{f^2 + F_N^2} = \sqrt{(275\text{ N})^2 + (833\text{ N})^2} = 877\text{ N}.$$

81. (a) If we choose "downhill" positive, then Newton's law gives

$$m_A g \sin\theta - f_A - T = m_A a$$

for block A (where θ = 30º). For block B we choose leftward as the positive direction and write $T - f_B = m_B a$. Now

$$f_A = \mu_{k,\text{incline}} F_{NA} = \mu' m_A g \cos\theta$$

using Eq. 6-12 applies to block A, and

$$f_B = \mu_k F_{NB} = \mu_k m_B g.$$

In this particular problem, we are asked to set μ' = 0, and the resulting equations can be straightforwardly solved for the tension: T = 13 N.

(b) Similarly, finding the value of a is straightforward:

$$a = g(m_A \sin\theta - \mu_k m_B)/(m_A + m_B) = 1.6\text{ m/s}^2.$$

85. The mass of the car is m = (10700/9.80) kg = 1.09×10^3 kg. We choose "inward" (horizontally towards the center of the circular path) as the positive direction.

(a) With v = 13.4 m/s and R = 61 m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = 3.21 \times 10^3\text{ N}.$$

(b) Noting that $F_N = mg$ in this situation, the maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700\text{ N}) = 3.75 \times 10^3\text{ N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

91. We apply Newton's second law (as $F_{\text{push}} - f = ma$). If we find $F_{\text{push}} < f_{\max}$, we conclude "no, the cabinet does not move" (which means a is actually 0 and $f = F_{\text{push}}$), and

if we obtain $a > 0$ then it is moves (so $f = f_k$). For f_{max} and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force equal to 556 N. Thus, f_{max} = 378 N and f_k = 311 N.

(a) Here we find $F_{push} < f_{max}$ which leads to $f = F_{push}$ = 222 N.

(b) Again we find $F_{push} < f_{max}$ which leads to $f = F_{push}$ = 334 N.

(c) Now we have $F_{push} > f_{max}$ which means it moves and $f = f_k$ = 311 N.

(d) Again we have $F_{push} > f_{max}$ which means it moves and $f = f_k$ = 311 N.

(e) The cabinet moves in (c) and (d).

99. Replace f_s with f_k in Fig. 6-5(b) to produce the appropriate force diagram for the first part of this problem (when it is sliding downhill with zero acceleration). This amounts to replacing the static coefficient with the kinetic coefficient in Eq. 6-13: $\mu_k = \tan\theta$. Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed from what is shown in Fig. 6-5(b). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$-mg\sin\theta - \mu_k mg\cos\theta = ma.$$

Canceling the mass and substituting what we found earlier for the coefficient, we have

$$-g\sin\theta - \tan\theta\, g\cos\theta = a\ .$$

This simplifies to $-2g\sin\theta = a$. Eq. 2-16 then gives the distance to stop: $\Delta x = -v_o^2/2a$.

(a) Thus, the distance up the incline traveled by the block is $\Delta x = v_o^2/(4g\sin\theta)$.

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in section 6-1). Sample Problem 6-2 treats the "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill): $\mu_s = \tan(\theta_{repose})$. Therefore, we expect $\theta_{repose} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

105. Probably the most appropriate picture in the textbook to represent the situation in this problem is in the previous chapter: Fig. 5-9. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as P (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces

$$P - f_k = ma$$
$$F_N - mg = 0.$$

Using $v^2 = v_0^2 + 2a\Delta x$ we find a = 0.36 m/s^2. Consequently, we obtain f_k = 71 N and F_N = 392 N. Therefore, $\mu_k = f_k / F_N$ = 0.18.

3. (a) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times10^7 \text{ m/s}\right)^2 + 2\ \left(3.6\times10^{15} \text{ m/s}^2\right)\left(0.035 \text{ m}\right)} = 2.9\times10^7 \text{ m/s.}$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}\ \left(1.67\times10^{-27} \text{ kg}\right)\left(2.4\times10^7 \text{ m/s}\right)^2 = 4.8\times10^{-13} \text{ J.}$$

The final kinetic energy is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\ \left(1.67\times10^{-27} \text{ kg}\right)\left(2.9\times10^7 \text{ m/s}\right)^2 = 6.9\times10^{-13} \text{ J.}$$

The change in kinetic energy is $\Delta K = 6.9 \times 10^{-13}$ J – 4.8×10^{-13} J = 2.1×10^{-13} J.

17. (a) We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is *mg* downward. Furthermore, the acceleration of the astronaut is *g*/10 upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11\ mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})}{10} = 1.164\times10^4\ \text{J}$$

which (with respect to significant figures) should be quoted as 1.2×10^4 J.

(b) The force of gravity has magnitude *mg* and is opposite in direction to the displacement. Thus, using Eq. 7-7, the work done by gravity is

$$W_g = -mgd = -\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = -1.058\times10^4\ \text{J}$$

which should be quoted as -1.1×10^4 J.

(c) The total work done is $W = 1.164\times10^4\text{ J} - 1.058\times10^4\text{ J} = 1.06\times10^3\text{ J}$. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to 1.1×10^3 J) is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(1.06\times10^3\text{ J})}{72\text{ kg}}}=5.4\text{ m/s}.$$

19. (a) We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude Mg). The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}}=m\vec{a}\Rightarrow\; Mg-F\;=\;M\left(\frac{g}{4}\right)$$

so $F = 3Mg/4$. The displacement is downward, so the work done by the cord's force is, using Eq. 7-7,

$$W_F = -Fd = -3Mgd/4.$$

(b) The force of gravity is in the same direction as the displacement, so it does work $W_g = Mgd$.

(c) The total work done on the block is $-3M\,gd/4+M\,gd\;=\;M\,gd/4$. Since the block starts from rest, we use Eq. 7-15 to conclude that this $(M\,gd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) Since $K=\frac{1}{2}Mv^2$, the speed is

$$v=\sqrt{\frac{2K}{M}}=\sqrt{\frac{2(Mgd/4)}{M}}=\sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

29. (a) As the body moves along the x axis from x_i = 3.0 m to x_f = 4.0 m the work done by the force is

$$W=\int_{x_i}^{x_f}F_x\;dx=\int_{x_i}^{x_f}-6x\;dx=-3(x_f^2-x_i^2)=-3\;(4.0^2-3.0^2)=-21\text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W=\Delta K=\frac{1}{2}m\left(v_f^2-v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21\text{ J})}{2.0\text{ kg}} + (8.0\text{ m/s})^2} = 6.6\text{ m/s}.$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$-3\left(x_f^2 - x_i^2\right) = \frac{1}{2} m \left(v_f^2 - v_i^2\right).$$

Thus,

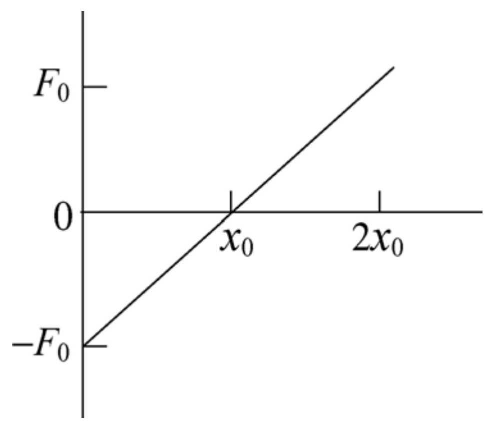
$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0\text{ kg}}{6\text{ N/m}}\left((5.0\text{ m/s})^2 - (8.0\text{ m/s})^2\right) + (3.0\text{ m})^2} = 4.7\text{ m}.$$

35. (a) The graph shows F as a function of x assuming x_0 is positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.

Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $-(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$(2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum, which is zero.



(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right) dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

43. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\text{ N})(5.0\text{ m/s})\cos 37° = 4.9 \times 10^2\text{ W}.$$

45. (a) The power is given by $P = Fv$ and the work done by $\vec{F}$ from time t_1 to time t_2 is given by

$$W = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} Fv\,dt.$$

Since $\vec{F}$ is the net force, the magnitude of the acceleration is $a = F/m$, and, since the initial velocity is $v_0 = 0$, the velocity as a function of time is given by $v = v_0 + at = (F/m)t$. Thus

$$W = \int_{t_1}^{t_2} (F^2/m)t\ \mathrm{d}t = \frac{1}{2}(F^2/m)(t_2^2 - t_1^2).$$

For $t_1 = 0$ and $t_2 = 1.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\ \mathrm{N})^2}{15\ \mathrm{kg}}\right)(1.0\ \mathrm{s})^2 = 0.83\ \mathrm{J}.$$

(b) For $t_1 = 1.0\,\mathrm{s}$, and $t_2 = 2.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\ \mathrm{N})^2}{15\ \mathrm{kg}}\right)[(2.0\ \mathrm{s})^2 - (1.0\ \mathrm{s})^2] = 2.5\ \mathrm{J}.$$

(c) For $t_1 = 2.0\,\mathrm{s}$ and $t_2 = 3.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\ \mathrm{N})^2}{15\ \mathrm{kg}}\right)[(3.0\ \mathrm{s})^2 - (2.0\ \mathrm{s})^2] = 4.2\ \mathrm{J}.$$

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2\, t/m$ for the power at any time t. At the end of the third second

$$P = \left(\frac{(5.0\ \mathrm{N})^2\ (3.0\ \mathrm{s})}{15\ \mathrm{kg}}\right) = 5.0\ \mathrm{W}.$$

47. The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W_T = W_e + W_c + W_s.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero. This means $W_e + W_c + W_s = 0$. The elevator moves upward through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(54\ \mathrm{m}) = -6.35 \times 10^5\ \mathrm{J}.$$

The counterweight moves downward the same distance, so the work done by gravity on it is

$$W_c = m_c gd = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03\times10^5 \text{ J}.$$

Since W_T = 0, the work done by the motor on the system is

$$W_s = -W_e - W_c = 6.35\times10^5 \text{ J} - 5.03\times10^5 \text{ J} = 1.32\times10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s},$ so the power supplied by the motor to lift the elevator is

$$P = \frac{W_s}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

63. (a) In 10 min the cart moves

$$d = \left(6.0 \ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft/mi}}{60 \text{ min/h}}\right)(10 \text{ min}) = 5280 \text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40 \text{ lb})(5280 \text{ ft}) \cos 30° = 1.8\times10^5 \text{ ft}\cdot\text{lb}.$$

(b) The average power is given by Eq. 7-42, and the conversion to horsepower (hp) can be found on the inside back cover. We note that 10 min is equivalent to 600 s.

$$P_{\text{avg}} = \frac{1.8\times10^5 \text{ ft} \cdot \text{lb}}{600 \text{ s}} = 305 \text{ ft}\cdot\text{lb/s}$$

which (upon dividing by 550) converts to P_{avg} = 0.55 hp.

69. (a) Eq. 7-6 gives $W_a = Fd$ = (209 N)(1.50 m) $\approx$ 314 J.

(b) Eq. 7-12 leads to W_g = (25.0 kg)(9.80 m/s^2)(1.50 m)cos(115º) ≈ –155 J.

(c) The angle between the normal force and the direction of motion remains 90º at all times, so the work it does is zero.

(d) The total work done on the crate is W_T = 314 J – 155 J =158 J.

71. (a) Hooke's law and the work done by a spring is discussed in the chapter. Taking absolute values, and writing that law in terms of differences ΔF and Δx, we analyze the first two pictures as follows:

$$\begin{aligned} |\Delta F| &= k|\Delta x| \\ 240\text{ N} - 110\text{ N} &= k(60\text{ mm} - 40\text{ mm}) \end{aligned}$$

which yields k = 6.5 N/mm. Designating the relaxed position (as read by that scale) as x_o we look again at the first picture:

$$110\text{ N} = k(40\text{ mm} - x_o)$$

which (upon using the above result for k) yields x_o = 23 mm.

(b) Using the results from part (a) to analyze that last picture, we find

$$W = k(30\,\text{mm} - x_o) = 45\text{ N}\ .$$

73. A convenient approach is provided by Eq. 7-48.

$$P = F\,v = (1800\text{ kg} + 4500\text{ kg})(9.8\text{ m/s}^2)(3.80\text{ m/s}) = 235\text{ kW}.$$

Note that we have set the applied force equal to the weight in order to maintain constant velocity (zero acceleration).

77. (a) We can easily fit the curve to a concave-downward parabola: $x = \frac{1}{10}t(10 - t)$, from which (by taking two derivatives) we find the acceleration to be a = –0.20 m/s^2. The (constant) force is therefore $F = ma$ = –0.40 N, with a corresponding work given by $W = Fx = \frac{2}{50}t(t - 10)$. It also follows from the x expression that v_o = 1.0 m/s. This means that $K_i = \frac{1}{2}mv^2$ = 1.0 J. Therefore, when t = 1.0 s, Eq. 7-10 gives $K = K_i + W = 0.64\text{ J} \approx 0.6\text{ J}$, where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives K = 0.

(c) Evaluating the $W = \frac{2}{50}t(t - 10)$ expression at t = 5.0 s and t = 1.0 s, and subtracting, yields –0.6 J. This can also be inferred from the answers for parts (a) and (b).

5. The potential energy stored by the spring is given by $U = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\,\text{J})}{(0.075\,\text{m})^2} = 8.9 \times 10^3\ \text{N/m}.$$

9. We neglect any work done by friction. We work with SI units, so the speed is converted: v = 130(1000/3600) = 36.1 m/s.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad h = \frac{v^2}{2g} = \frac{(36.1\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66.5\ \text{m}\ .$$

If L is the length of the ramp, then L sin 15° = 66.5 m so that L = 66.5/sin 15° = 257 m. Therefore, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

11. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$.

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where r = 0.220 m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f. Energy conservation leads to

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\,(1 - \cos\theta)\ .$$

The speed is $v = \sqrt{2gr} = 2.08\ \text{m/s}$.

(b) Since the expression for speed does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

31. We refer to its starting point as *A*, the point where it first comes into contact with the spring as *B*, and the point where the spring is compressed $|x| = 0.055$ m as *C*. Point *C* is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270\ \text{N}}{0.02\ \text{m}} = 1.35 \times 10^4\ \text{N/m}\ .$$

(a) The distance between points *A* and *B* is $\vec{F}_g$ and we note that the total sliding distance $\ell + |x|$ is related to the initial height *h* of the block (measured relative to *C*) by

$$\frac{h}{\ell + |x|} = \sin\theta$$

where the incline angle θ is 30°. Mechanical energy conservation leads to

$$\begin{aligned} K_A + U_A &= K_C + U_C \\ 0 + mgh &= 0 + \frac{1}{2}kx^2 \end{aligned}$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4\ \text{N/m})(0.055\ \text{m})^2}{2(12\ \text{kg})\ (9.8\ \text{m/s}^2)} = 0.174\ \text{m}\ .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30°} = \frac{0.174\ \text{m}}{\sin 30°} = 0.35\ \text{m}\ .$$

(b) From this result, we find $\ell = 0.35 - 0.055 = 0.29\ \text{m}$, which means that $\Delta y = -\ell \sin\theta = -0.15\ \text{m}$ in sliding from point *A* to point *B*. Thus, Eq. 8-18 gives

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}mv_B^2 + mg\Delta h = 0$$

which yields $v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7\ \text{m/s}$.

45. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\,\text{J}.$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\,\text{J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x and the y component of Newton's second law are

$$x:\quad F\cos\theta - f = 0$$
$$y:\quad F_N + F\sin\theta - mg = 0,$$

where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F\cos\theta = (7.68\ \text{N})\cos 15.0° = 7.42\ \text{N}$$

and the second gives

$$F_N = mg - F\sin\theta = (3.57\ \text{kg})(9.8\ \text{m/s}^2) - (7.68\ \text{N})\sin 15.0° = 33.0\ \text{N}.$$

Thus,

$$\mu_k = \frac{f}{F_N} = \frac{7.42\ \text{N}}{33.0\ \text{N}} = 0.225\,.$$

47. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25\ \text{kg})\left(9.8\ \text{m/s}^2\right)(12\ \text{m}) = -2.9\times10^3\ \text{J}\,.$$

(b) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(25\ \text{kg})(5.6\ \text{m/s})^2 = 3.9\times10^2\ \text{J}$.

(c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9\times10^2 \text{ J} - 2.9\times10^3 \text{ J}}{12 \text{ m}} = 2.1\times10^2 \text{ N}$$

69. There is the same potential energy change in both circumstances, so we can equate the kinetic energy changes as well:

$$\Delta K_2 = \Delta K_1 \Rightarrow \frac{1}{2} m\, v_B^2 - \frac{1}{2} m(4.00)^2 = \frac{1}{2} m(2.60)^2 - \frac{1}{2} m(2.00)^2$$

which leads to v_B = 4.33 m/s.

75. We note that if the larger mass (block B, m_B = 2 kg) falls d = 0.25 m, then the smaller mass (blocks A, m_A = 1 kg) must increase its height by $h = d\sin 30°$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = m_B g d - m_A g h = 3.7 \text{ J}.$$

83. The initial height shown in the figure is the y = 0 level in our computations of U_g, and in parts (a) and (b) the heights are y_a = 0.80 sin 40° = 0.51 m and y_b = 1.00 sin 40° = 0.64 m, respectively.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_a + U_a \Rightarrow 16 + 0 = K_a + mgy_a + \frac{1}{2}k(0.20)^2$$

from which we obtain K_a = 16 – 5.0 – 4.0 = 7.0 J.

(b) Again we use the conservation of energy

$$K_i + U_i = K_b + U_b \Rightarrow K_i + 0 = 0 + mgy_b + \frac{1}{2} k(0.40)^2$$

from which we obtain K_i = 6.0 + 16 = 22 J.

87. Since the speed is constant ΔK = 0 and Eq. 8-33 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th(cube)}} + \Delta E_{\text{th(floor)}}.$$

Thus, if W_{applied} = (15)(3.0) = 45 J, and we are told that $\Delta E_{\text{th (cube)}}$ = 20 J, then we conclude that $\Delta E_{\text{th (floor)}}$ = 25 J.

109. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the

gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mg\,(2L) = \frac{1}{2}mv^2 + 0$$

This leads to $v = 2\sqrt{gL}$. With $L = 0.62$ m, we have

$$v = 2\sqrt{(9.8\ \text{m/s}^2)(0.62\ \text{m})} = 4.9\ \text{m/s}\,.$$

(b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\,mg.$$

With $m = 0.092$ kg, the tension is given by $T = 4.5$ N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, then Newton's second law applied to the axis along the rod yields

$$T - mg\cos\theta = m\frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$K_i + U_i = K + U$$
$$0 + mgL = \frac{1}{2}mv^2 + mgL\,(1 - \cos\theta)$$
$$gL = \frac{1}{2}(gL(1 - \cos\theta)) + gL\,(1 - \cos\theta)$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 71°\,.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

111. (a) At the top of its flight, the vertical component of the velocity vanishes, and the horizontal component (neglecting air friction) is the same as it was when it was thrown. Thus,

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.050\,\text{kg})\left((8.0\,\text{m/s})\cos 30°\right)^2 = 1.2\text{ J.}$$

(b) We choose the point 3.0 m below the window as the reference level for computing the potential energy. Thus, equating the mechanical energy when it was thrown to when it is at this reference level, we have (with SI units understood)

$$mgy_0 + K_0 = K$$
$$m(9.8)(3.0) + \frac{1}{2}m(8.0)^2 = \frac{1}{2}mv^2$$

which yields (after canceling m and simplifying) v = 11 m/s.

(c) As mentioned, m cancels — and is therefore not relevant to that computation.

(d) The v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on the direction.

119. (a) During the final d = 12 m of motion, we use

$$K_1 + U_1 = K_2 + U_2 + f_k d$$
$$\frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

where v = 4.2 m/s. This gives f_k = 0.31 N. Therefore, the thermal energy change is $f_k d = 3.7$ J.

(b) Using f_k = 0.31 N we obtain $f_k d_{\text{total}}$ = 4.3 J for the thermal energy generated by friction; here, d_{total} = 14 m.

(c) During the initial d' = 2 m of motion, we have

$$K_0 + U_0 + W_{\text{app}} = K_1 + U_1 + f_k d' \Rightarrow 0 + 0 + W_{\text{app}} = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-33. This leads to the result W_{app} = 4.3 J, and — reasonably enough — is the same as our answer in part (b).

121. We use Eq. 8-20.

(a) The force at x = 2.0 m is

$$F = -\frac{dU}{dx} \approx -\frac{-(17.5)-(-2.8)}{4.0-1.0} = 4.9\,\text{N}.$$

(b) The force points in the +x direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(c) The total mechanical energy at x = 2.0 m is

$$E = \frac{1}{2}mv^2 + U \approx \frac{1}{2}(2.0)(-1.5)^2 - 7.7 = -5.5$$

in SI units (Joules). Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (–5.5 J) on the graph, we find two points where the potential energy curve has that value — at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m. The left boundary is at x = 1.5 m.

(d) From the above results, the right boundary is at x = 13.5 m.

(e) At x = 7.0 m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12\text{ J} \Rightarrow v = \sqrt{\frac{2}{m}(E-U)} \approx 3.5\text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

123. Converting to SI units, $v_0 = 8.3$ m/s and $v = 11.1$ m/s. The incline angle is $\theta = 5.0°$. The height difference between the car's highest and lowest points is (50 m) sin θ = 4.4 m. We take the lowest point (the car's final reported location) to correspond to the y = 0 reference level.

(a) Using Eq. 8-31 and Eq. 8-33, we find

$$f_k d = -\Delta K - \Delta U \;\Rightarrow\; f_k d = \frac{1}{2}m\left(v_0^2 - v^2\right) + mgy_0\ .$$

Therefore, the mechanical energy reduction (due to friction) is $f_k d = 2.4 \times 10^4$ J.

(b) With d = 50 m, we solve for f_k and obtain 4.7×10^2 N.

127. (a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2} m \left(v^2 - v_0^2\right) .$$

Therefore, $\Delta E = 6.0 \times 10^3$ J .

(b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W} .$$

(c) and (d) The constant applied force is ma = 30 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

131. The power generation (assumed constant, so average power is the same as instantaneous power) is

$$P = \frac{mgh}{t} = \frac{(3/4)(1200\,\text{m}^3)(10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(100\,\text{m})}{1.0\,\text{s}} = 8.80 \times 10^8 \text{ W}.$$

133. (a) Sample Problem 8-3 illustrates simple energy conservation in a similar situation, and derives the frequently encountered relationship: v = $\sqrt{2gh}$. In our present problem, the height change is equal to the rod length L. Thus, using the suggested notation for the speed, we have $v_o = \sqrt{2gL}$.

(b) At B the speed is (from Eq. 8-17)

$$v = \sqrt{v_0^2 + 2gL} = \sqrt{4gL} .$$

The direction of the centripetal acceleration ($v^2/r = 4gL/L = 4g$) is upward (at that moment), as is the tension force. Thus, Newton's second law gives

$$T - mg = m(4g) \Rightarrow T = 5mg.$$

(c) The difference in height between C and D is L, so the "loss" of mechanical energy (which goes into thermal energy) is $-mgL$.

(d) The difference in height between B and D is $2L$, so the total "loss" of mechanical energy (which all goes into thermal energy) is $-2mgL$.

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the +x axis is rightward, and the +y direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60° = 15.3\text{ m.}$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60° = 20\text{ m/s.}$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m.}$$

23. The initial direction of motion is in the +x direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4\text{ N}\cdot\text{s}}{2.70\times10^{-2}\text{ s}} = 1.20\times10^3\text{ N}$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{avg}\Delta t = mv_f - mv_i.$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. Thus,

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times10^{-3}\,\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

(a) The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20\times10^3$ N .

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

35. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$\begin{aligned}J &= \int_0^{3.0\times10^{-3}} F\,dt = \int_0^{3.0\times10^{-3}} \left[(6.0\times10^6)\,t - (2.0\times10^9)t^2\right]dt \\ &= \left[\frac{1}{2}(6.0\times10^6)t^2 - \frac{1}{3}(2.0\times10^9)t^3\right]\Bigg|_0^{3.0\times10^{-3}} \\ &= 9.0\,\text{N}\cdot\text{s}.\end{aligned}$$

(b) Since $J = F_{\text{avg}}\,\Delta t$, we find

$$F_{\text{avg}}\ \frac{J}{\Delta t} = \frac{9.0\ \text{N}\cdot\text{s}}{3.0\times10^{-3}\ \text{s}} = 3.0\times10^3\ \text{N}.$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t. The result is $t = 1.5\times10^{-3}$ s. At that time the force is

$$F_{\text{max}} = (6.0\times10^6)(1.5\times10^{-3}) - (2.0\times10^9)(1.5\times10^{-3})^2 = 4.5\times10^3\ \text{N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v its speed as it leaves the foot. Then,

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\ \text{N}\cdot\text{s}}{0.45\ \text{kg}} = 20\ \text{m/s}.$$

39. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then

$$m_s v_s + m_m v_m = 0 \rightarrow \; v_m = -m_s v_s / m_m.$$

We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068 \text{ kg})(4.0 \text{ m/s})}{91 \text{ kg}} = -3.0\times10^{-3} \text{ m/s},$$

or $|v_m| = 3.0\times10^{-3}$ m/s. The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

47. Our notation is as follows: the mass of the original body is M = 20.0 kg; its initial velocity is $\vec{v}_0 = 200\hat{\text{i}}$ in SI units (m/s); the mass of one fragment is m_1 = 10.0 kg; its velocity is $\vec{v}_1 = -100\hat{\text{j}}$ in SI units; the mass of the second fragment is m_2 = 4.0 kg; its velocity is $\vec{v}_2 = -500\hat{\text{i}}$ in SI units; and, the mass of the third fragment is m_3 = 6.00 kg.

(a) Conservation of linear momentum requires $M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$, which (using the above information) leads to

$$\vec{v}_3 = (1.00\times10^3\,\hat{\text{i}} - 0.167\times10^3\,\hat{\text{j}}) \text{ m/s}$$

in SI units. The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{1000^2 + (-167)^2} = 1.01\times10^3$ m/s. It points at $\tan^{-1}$ (–167/1000) = –9.48° (that is, at 9.5° measured clockwise from the +x axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23\times10^6 \text{ J}.$$

61. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}.$$

Using SI units (so m_1 = 0.34 kg), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2 \text{ m/s} - 0.66 \text{ m/s}}{1.2 \text{ m/s} + 0.66 \text{ m/s}} \right) (0.34 \text{ kg}) = 0.099 \text{ kg}.$$

(b) The velocity of the second cart is given by Eq. 9-68:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34 \text{ kg})}{0.34 \text{ kg} + 0.099 \text{ kg}} \right) (1.2 \text{ m/s}) = 1.9 \text{ m/s}.$$

(c) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93 \text{ m/s}.$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

63. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \; .$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 \; .$$

We combine this with $v_{1f} = v_{1i} / 4$ to obtain $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2 \text{ kg}$.

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s}.$$

77. (a) The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. For this problem $R = 480$ kg/s and $v_{\text{rel}} = 3.27 \times 10^3$ m/s, so

$$T = (480 \text{ kg/s})(3.27 \times 10^3 \text{ m/s}) = 1.57 \times 10^6 \text{ N}.$$

(b) The mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, M_{fuel} = (480 kg/s)(250 s) = 1.20×10^5 kg. The mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5 \text{ kg}) - (1.20 \times 10^5 \text{ kg}) = 1.35 \times 10^5 \text{ kg}.$$

(c) Since the initial speed is zero, the final speed is given by

$$v_f = v_{\text{rel}} \ln\frac{M_i}{M_f} = \left(3.27\times10^3\right)\ln\left(\frac{2.55\times10^5}{1.35\times10^5}\right) = 2.08\times10^3 \text{ m/s}.$$

79. (a) We consider what must happen to the coal that lands on the faster barge during one minute (Δt = 60s). In that time, a total of m = 1000 kg of coal must experience a change of velocity $\Delta v = 20\,\text{km/h} - 10\,\text{km/h} = 10\,\text{km/h} = 2.8\,\text{m/s}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = \frac{(1000 \text{ kg})(2.8 \text{ m/s})}{60 \text{ s}} = 46\,\text{N}$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$.

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

91. (a) If m is the mass of a pellet and v is its velocity as it hits the wall, then its momentum is $p = mv = (2.0 \times 10^{-3}$ kg)(500 m/s) = 1.0 kg · m/s, toward the wall.

(b) The kinetic energy of a pellet is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.0\times10^{-3}\text{kg}\right)\left(500\,\text{m/s}\right)^2 = 2.5\times10^2\,\text{J}\,.$$

(c) The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers p = 1.0 kg · m/s. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred is

$$F_{\text{avg}} = \frac{p\Delta N}{\Delta t} = \left(1.0\,\text{kg}\cdot\text{m/s}\right)\left(10\,\text{s}^{-1}\right) = 10\,\text{N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If Δt is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F_{\text{avg}} = \frac{p}{\Delta t} = \frac{1.0\,\text{kg}\cdot\text{m/s}}{0.6\times10^{-3}\,\text{s}} = 1.7\times10^{3}\,\text{N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

93. (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\hat{\text{j}} = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{j}}$$

and the final momentum is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$. The impulse on it equals the change in its momentum: $\vec{J} = \vec{p}_f - \vec{p}_i = (7400\,\text{N}\cdot\text{s})(\hat{\text{i}} - \hat{\text{j}})$.

(b) The initial momentum of the car is $\vec{p}_i = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ and the final momentum is $\vec{p}_f = 0$. The impulse acting on it is $\vec{J} = \vec{p}_f - \vec{p}_i = (-7.4\times10^{3}\,\text{N}\cdot\text{s})\hat{\text{i}}$.

(c) The average force on the car is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\text{i}} - \hat{\text{j}})}{4.6\,\text{s}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$$

and its magnitude is $F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2.3\times10^{3}\,\text{N}$.

(d) The average force is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}}{350\times10^{-3}\,\text{s}} = (-2.1\times10^{4}\,\text{N})\hat{\text{i}}$$

and its magnitude is $F_{\text{avg}} = 2.1 \times 10^4$ N.

(e) The average force is given above in unit vector notation. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.

97. Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to $m_F v_F = (m_F + m_C)v$, so $v = v_F m_F/(m_F + m_C)$. The initial kinetic energy of the system is

$$K_i = \frac{1}{2} m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Thus,

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2} m_F v_F^2\right).$$

Simplifying, we obtain $m_F/(m_F + m_C) = 0.73,$ which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

101. The mass of each ball is m, and the initial speed of one of the balls is $v_{1i} = 2.2\,\text{m/s}$. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m v_{1i} &= m v_{1f} \cos\theta_1 + m v_{2f} \cos\theta_2 \\ 0 &= m v_{1f} \sin\theta_1 - m v_{2f} \sin\theta_2 \end{aligned}$$

The mass m cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

(a) The y-momentum equation can be rewritten as, using $\theta_2 = 60°$ and $v_{2f} = 1.1$ m/s,

$$v_{1f} \sin\theta_1 = (1.1 \text{ m/s}) \sin 60° = 0.95 \text{ m/s}.$$

and the x-momentum equation yields

$$v_{1f}\cos\theta_1 = (2.2 \text{ m/s}) - (1.1 \text{ m/s})\cos 60° = 1.65 \text{ m/s}.$$

Dividing these two equations, we find $\tan\theta_1 = 0.576$ which yields $\theta_1 = 30°$. We plug the value into either equation and find $v_{1f} \approx 1.9$ m/s.

(b) From the above, we have $\theta_1 = 30°$.

(c) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_{1i}^2 \quad \text{and} \quad \frac{2K_f}{m} = v_{1f}^2 + v_{2f}^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed "exactly" as $v_{1f} = \frac{1}{2}v_{1i}\sqrt{3}$ (and of course $v_{2f} = \frac{1}{2}v_{1i}$ "exactly" — which makes it clear that these two kinetic energy expressions are indeed equal).

107. (a) Noting that the initial velocity of the system is zero, we use Eq. 9-19 and Eq. 2-15 (adapted to two dimensions) to obtain

$$\vec{d} = \frac{1}{2}\left(\frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2}\right)t^2 = \frac{1}{2}\left(\frac{-2\hat{\text{i}} + \hat{\text{j}}}{0.006}\right)(0.002)^2$$

which has a magnitude of 0.745 mm.

(b) The angle of $\vec{d}$ is 153° counterclockwise from $+x$-axis.

(c) A similar calculation using Eq. 2-11 (adapted to two dimensions) leads to a center of mass velocity of $\vec{\text{v}}$ = 0.7453 m/s at 153°. Thus, the center of mass kinetic energy is

$$K_{\text{com}} = \frac{1}{2}(m_1 + m_2)v^2 = 0.00167 \text{ J}.$$

113. By conservation of momentum, the final speed v of the sled satisfies

$$(2900 \text{ kg})(250 \text{ m/s}) = (2900 \text{ kg} + 920 \text{ kg})v$$

which gives $v = 190$ m/s.

115. (a) We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_M}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{\text{com}} = \frac{(7.36\times10^{22}\ \text{kg})(3.82\times10^{8}\ \text{m})}{7.36\times10^{22}\ \text{kg} + 5.98\times10^{24}\ \text{kg}} = 4.64\times10^{6}\ \text{m} \approx 4.6\times10^{3}\ \text{km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6$ m, so $r_{\text{com}} / R_E = 0.73 = 73\%$.

117. (a) The thrust is Rv_{rel} where $v_{\text{rel}} = 1200$ m/s. For this to equal the weight Mg where $M = 6100$ kg, we must have $R = (6100)\ (9.8)/1200 \approx 50$ kg/s.

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\text{rel}} - Mg = Ma$$

so that requiring $a = 21$ m/s^2 leads to $R = (6100)(9.8 + 21)/1200 = 1.6 \times 10^2$ kg/s.

129. Using Eq. 9-68 with $m_1 = 3.0$ kg, $v_{1i} = 8.0$ m/s and $v_{2f} = 6.0$ m/s, then

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \quad \Rightarrow \quad m_2 = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right)$$

leads to $m_2 = M = 5.0$ kg.

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0$ s interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t} = \frac{120 \text{ rad } - \frac{1}{2}\left(3.0 \text{ rad/s}^2\right)\left(4.0 \text{ s}\right)^2}{4.0 \text{ s}} = 24 \text{ rad/s}.$$

We now use $\omega = \omega_0 + \alpha t$ (Eq. 10-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24 \text{ rad / s}}{3.0 \text{ rad / s}^2} = -8.0 \text{ s}.$$

That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

21. (a) We obtain

$$\omega = \frac{(200 \text{ rev / min})(2\pi \text{ rad / rev})}{60 \text{ s / min}} = 20.9 \text{ rad / s}.$$

(b) With $r = 1.20/2 = 0.60$ m, Eq. 10-18 leads to $v = r\omega = (0.60)(20.9) = 12.5$ m/s.

(c) With $t = 1$ min, $\omega = 1000$ rev/min and $\omega_0 = 200$ rev/min, Eq. 10-12 gives

$$\alpha = \frac{\omega - \omega_o}{t} = 800 \text{ rev / min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(200 + 100)(1) = 600 \text{ rev}.$$

29. (a) Earth makes one rotation per day and 1 d is (24 h) (3600 s/h) = 8.64×10^4 s, so the angular speed of Earth is

$$\omega = \frac{2\pi \text{ rad}}{8.64 \times 10^4 \text{ s}} = 7.3 \times 10^{-5} \text{ rad/s}.$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R \cos 40°$, where R is the radius of Earth (6.4×10^6 m). Therefore, its speed is

$$v = \omega(R \cos 40°) = (7.3 \times 10^{-5} \text{ rad/s})(6.4 \times 10^6 \text{ m})\cos 40° = 3.5 \times 10^2 \text{ m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = (7.3 \times 10^{-5}\ \text{rad/s})(6.4 \times 10^{6}\ \text{m}) = 4.6 \times 10^{2}\ \text{m/s}.$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2} I\omega^2$, where I is the rotational inertia (in $\text{kg} \cdot \text{m}^2$) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602\ \text{rev / min})(2\pi\ \text{rad / rev})}{60\ \text{s / min}} = 63.0\ \text{rad / s}.$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400\ \text{J})}{(63.0\ \text{rad / s})^2} = 12.3\ \text{kg} \cdot \text{m}^2.$$

35. We use the parallel axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies h = 0.50 m – 0.20 m = 0.30 m. We find

$$I_{\text{com}} = \frac{1}{12} ML^2 = \frac{1}{12}(0.56\ \text{kg})(1.0\ \text{m})^2 = 4.67 \times 10^{-2}\ \text{kg} \cdot \text{m}^2.$$

Consequently, the parallel axis theorem yields

$$I = 4.67 \times 10^{-2}\ \text{kg} \cdot \text{m}^2 + (0.56\ \text{kg})(0.30\ \text{m})^2 = 9.7 \times 10^{-2}\ \text{kg} \cdot \text{m}^2.$$

37. Since the rotational inertia of a cylinder is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{4} MR^2\omega^2.$$

(a) For the smaller cylinder, we have $K = \frac{1}{4}(1.25)(0.25)^2(235)^2 = 1.1 \times 10^3$ J.

(b) For the larger cylinder, we obtain $K = \frac{1}{4}(1.25)(0.75)^2(235)^2 = 9.7 \times 10^3$ J.

41. We use the parallel-axis theorem. According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}\left(a^2 + b^2\right).$$

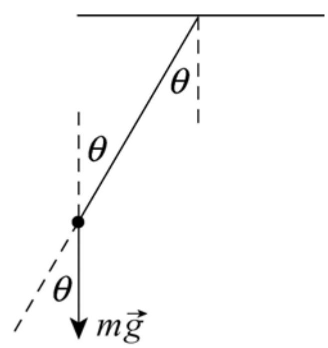
A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

$$I = I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2+b^2\right) + \frac{M}{4}\left(a^2+b^2\right) = \frac{M}{3}\left(a^2+b^2\right)$$
$$= \frac{0.172\ \text{kg}}{3}[(0.035\ \text{m})^2 + (0.084\ \text{m})^2] = 4.7\times10^{-4}\ \text{kg}\cdot\text{m}^2.$$

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is $mg \sin \theta$. If ℓ is the length of the rod, then the torque associated with this force has magnitude

$$\tau = mg\ell\sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6\ \text{N}\cdot\text{m}\,.$$

For the position shown, the torque is counter-clockwise.



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47. We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin \theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2 F_2 \sin \theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2.$$

Substituting the given values, we obtain

$$\tau = (1.30\ \text{m})(4.20\ \text{N})\sin 75° - (2.15\ \text{m})(4.90\ \text{N})\sin 60° = -3.85\ \text{N}\cdot\text{m}.$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad/s}^2.$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = \left(12.0 \text{ kg}\cdot\text{m}^2\right)\left(28.2 \text{ rad/s}^2\right) = 3.38 \times 10^2 \text{ N}\cdot\text{m}.$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell,$ where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8 \text{ m/s}^2\right)\left(1.00 \text{ m}\right)} = 5.42 \text{ m/s}.$$

69. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

(a) From Eq. 10-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;\Rightarrow\; \alpha = \frac{2\theta}{t^2} = \frac{2(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 314 \text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 7.54 \text{ m/s}^2.$$

(c) From the first of the above equations, we find

$$T_1 = m_1\left(g - a_1\right) = M\left(g - \frac{2R\theta}{t^2}\right) = (6.20 \text{ kg})\left(9.80 \text{ m/s}^2 - \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2}\right) = 14.0 \text{ N}.$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = M\left(g - \frac{2R\theta}{t^2}\right) - \frac{2I\theta}{Rt^2} = 14.0 \text{ N} - \frac{(7.40\times10^{-4} \text{kg}\cdot\text{m}^2)(314 \text{ rad/s}^2)}{0.024 \text{ m}} = 4.36 \text{ N}.$$

87. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where $m = 2.0$ kg. Applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$, where $I = 0.050 \text{ kg}\cdot\text{m}^2$.

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \;\Rightarrow\; T = a\frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$.
Thus, $a = 0.92 \text{ m/s}^2$ and therefore $\alpha = -4.6 \text{ rad/s}^2$ (or $|\alpha| = 4.6 \text{ rad/s}^2$), where the negative sign in α should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

89. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50$ min. Thus,

$$\alpha = -\frac{33.33 \text{ rev/min}}{0.50 \text{min}} = -66.7 \text{ rev/min}^2 \approx -67 \text{ rev/min}^2.$$

(b) We use Eq. 10-13: $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2 = (33.33)(0.50) + \dfrac{1}{2}(-66.7)(0.50)^2 = 8.3$ rev.

91. (a) According to Table 10-2, the rotational inertia formulas for the cylinder (radius *R*) and the hoop (radius *r*) are given by

$$I_C = \frac{1}{2} MR^2 \;\text{ and }\; I_H = Mr^2.$$

Since the two bodies have the same mass, then they will have the same rotational inertia if

$$R^2/2 = R_H^2 \;\rightarrow\; R_H = R/\sqrt{2}\,.$$

(b) We require the rotational inertia to be written as $I = Mk^2$, where *M* is the mass of the given body and *k* is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

115. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2} m_{\text{box}} v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is $\omega = 1.41/0.20 = 0.71$ rad/s. Thus, the kinetic energy of rotation is $\frac{1}{2} I\omega^2 = 10.0$ J.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + 0 = (6.0 + 10.0) + m_{\text{box}} g(-h).$$

Therefore, $h = 16.0/58.8 = 0.27$ m.

5. By Eq. 10-52, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$ (Eq. 11-5), where $I = mR^2$ is its rotational inertia about the center of mass, m = 140 kg, and v = 0.150 m/s is the speed of its center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2} mR^2 \left(\frac{v^2}{R^2}\right) + \frac{1}{2} mv^2 = mv^2 = (140\text{ kg})(0.150\text{ m/s})^2$$

which implies that the work required is – 3.15 J.

23. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}},$ then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\hat{\text{i}} + (zF_x - xF_z)\hat{\text{j}} + (xF_y - yF_x)\hat{\text{k}}.$$

(a) Plugging in, we find $\vec{\tau} = \left[(3.0\text{m})(6.0\text{N}) - (4.0\text{m})(-8.0\text{N})\right]\hat{\text{k}} = (50\,\text{N}\cdot\text{m})\,\hat{\text{k}}.$

(b) We use Eq. 3-27, $|\vec{r} \times \vec{F}| = rF \sin\phi,$ where ϕ is the angle between $\vec{r}$ and $\vec{F}$. Now $r = \sqrt{x^2 + y^2} = 5.0\text{ m}$ and $F = \sqrt{F_x^2 + F_y^2} = 10\text{ N}.$ Thus,

$$rF = (5.0\text{ m})(10\text{ N}) = 50\text{ N}\cdot\text{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies $\sin\phi = 1$ and $\phi = 90°$.

29. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v},$ where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = (-xv_z + zv_z)\hat{\text{j}}.$ Therefore,

$$\vec{\ell} = m(-xv_z + zv_x)\hat{\text{j}} = (0.25\text{ kg})\big(-(2.0\text{ m})(5.0\text{ m/s}) + (-2.0\text{ m})(-5.0\text{ m/s})\big)\hat{\text{j}} = 0.$$

(b) If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}},$ then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\hat{\text{i}} + (zF_x - xF_z)\hat{\text{j}} + (xF_y - yF_x)\hat{\text{k}}.$$

With x = 2.0, z = –2.0, F_y = 4.0 and all other components zero (and SI units understood) the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(8.0\,\hat{\text{i}} + 8.0\,\hat{\text{k}}\right)\ \text{N}\cdot\text{m}.$$

33. If we write (for the general case) $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}},$ then (using Eq. 3-30) we find $\vec{r} \times \vec{v}$ is equal to

$$\left(yv_z - zv_y\right)\hat{\text{i}} + \left(zv_x - xv_z\right)\hat{\text{j}} + \left(xv_y - yv_x\right)\hat{\text{k}}.$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v},$ where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and m = 3.0 kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = \left(3.0\right)[(3.0)(-6.0) - (8.0)(5.0)]\hat{\text{k}} = (-1.7\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(b) The torque is given by Eq. 11-14, $\vec{\tau} = \vec{r} \times \vec{F}.$ We write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}}$ and $\vec{F} = F_x\hat{\text{i}}$ and obtain

$$\vec{\tau} = \left(x\hat{\text{i}} + y\hat{\text{j}}\right) \times \left(F_x\hat{\text{i}}\right) = -yF_x\hat{\text{k}}$$

since $\hat{\text{i}} \times \hat{\text{i}} = 0$ and $\hat{\text{j}} \times \hat{\text{i}} = -\hat{\text{k}}.$ Thus, we find

$$\vec{\tau} = -\left(8.0\,\text{m}\right)\left(-7.0\,\text{N}\right)\hat{\text{k}} = (56\,\text{N}\cdot\text{m})\hat{\text{k}}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt,$ so the rate of change of the angular momentum is 56 $\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

37. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{\text{avg}} = \left(L_f - L_i\right)/\Delta t,$ where L_i is the initial angular momentum and L_f is the final angular momentum. Thus,

$$\tau_{\text{avg}} = \frac{0.800\ \text{kg}\cdot\text{m}^2/\text{s} - 3.00\ \text{kg}\cdot\text{m}^2/\text{s}}{1.50\,\text{s}} = -1.47\ \text{N}\cdot\text{m},$$

or $|\tau_{\text{avg}}| = 1.47\ \text{N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \alpha t^2/2.$ If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau\, t^2/2}{I} = \frac{\left(3.00\,\mathrm{kg\cdot m^2/s}\right)\left(1.50\,\mathrm{s}\right) + \left(-1.467\,\mathrm{N\cdot m}\right)\left(1.50\,\mathrm{s}\right)^2/2}{0.140\,\mathrm{kg\cdot m^2}} = 20.4\,\mathrm{rad}.$$

(c) The work done on the wheel is

$$W = \tau\theta = \left(-1.47\,\mathrm{N\cdot m}\right)\left(20.4\,\mathrm{rad}\right) = -29.9\ \mathrm{J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 10-52, which, if desired, can be rewritten as $W = \left(L_f^2 - L_i^2\right)/2I$.

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

43. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ and

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\,\mathrm{kg\cdot m^2}}{2.0\,\mathrm{kg\cdot m^2}}\right)\left(1.2\,\mathrm{rev/s}\right) = 3.6\,\mathrm{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2} I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2} I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{\left(2.0\,\mathrm{kg\cdot m^2}\right)\left(3.6\,\mathrm{rev/s}\right)^2/2}{\left(6.0\,\mathrm{kg\cdot m^2}\right)\left(1.2\,\mathrm{rev/s}\right)^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

45. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning $\left(\text{at } \omega_i\right)$ and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1\,\omega_i = \left(I_1 + I_2\right)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800\ \mathrm{rev/min}$, we obtain $\omega_f = 267\ \mathrm{rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2} I_1 \omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6} I\omega_i^2.$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$ is

$$1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

49. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is

$$\vec{L}_f = I\omega\hat{k} = -MR^2|\omega|\hat{k},$$

where $\hat{k}$ is *up* in Fig. 11-47 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 11-47 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v - |\omega|R)R\hat{k}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\hat{k} + m(v - |\omega|R)R\hat{k}.$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{(0.15 \text{ m/s})}{(1.1+1)(0.43 \text{ m})} = 0.17 \text{ rad/s}.$$

67. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where $d = 0.500$ m is the length of the rod. The angular momentum of the wad is $mvd/2$ where $m = 0.0500$ kg and $v = 3.00$ m/s are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where

$$I = (2M + m)(d/2)^2$$

and M = 2.00 kg is the mass of each of the balls. We solve

$$mvd/2 = (2M + m)(d/2)^2\,\omega$$

for the angular speed:

$$\omega = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\text{ kg})(3.00\text{ m/s})}{(2(2.00\text{ kg}) + 0.0500\text{ kg})(0.500\text{ m})} = 0.148\text{ rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M + m} = \frac{0.0500\text{ kg}}{2(2.00\text{ kg}) + 0.0500\text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos\theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M + m)(d/2)^2\,\omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\frac{d}{2} + \frac{1}{2}(2M + m)\left(\frac{d}{2}\right)^2\omega^2 = mg\frac{d}{2}(1 - \cos\theta).$$

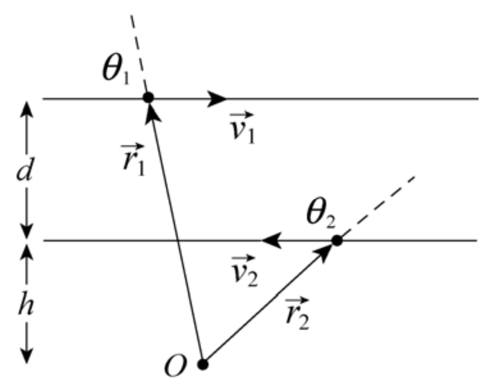
When this equation is solved for $\cos\theta$, the result is

$$\cos\theta = -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 = -\frac{1}{2}\left(\frac{2(2.00\text{ kg})+0.0500\text{ kg}}{(0.0500\text{ kg})(9.8\text{ m/s}^2)}\right)\left(\frac{0.500\text{ m}}{2}\right)(0.148\text{ rad/s})^2$$
$$= -0.0226.$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is 90° + 91.3° = 181°.

73. (a) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin\theta_1 = mv(d+h)$$



and it is into the page. The angular momentum of particle 2 has magnitude

$$\ell_2 = mvr_2 \sin\theta_2 = mvh$$

and it is out of the page. The net angular momentum has magnitude

$$\begin{aligned} L &= mv(d+h) - mvh = mvd \\ &= (2.90\times10^{-4}\text{ kg})(5.46\text{ m/s})(0.042\text{ m}) \\ &= 6.65\times10^{-5}\text{ kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

and is into the page. This result is independent of the location of the origin.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$L = mv(d + h) + mvh = mv(d + 2h).$$

This result depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

77. As the wheel-axel system rolls down the inclined plane by a distance d, the decrease in potential energy is $\Delta U = mgd\sin\theta$. This must be equal to the total kinetic energy gained:

$$mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd\sin\theta = \frac{1}{2}I\omega^2\left(\frac{mr^2}{I}+1\right) = K_{\text{rot}}\left(\frac{mr^2}{I}+1\right)$$

(a) With m=10.0 kg, d = 2.00 m, r = 0.200 m, and I = 0.600 kg m^2, mr^2/I =2/3, the rotational kinetic energy may be obtained as $98\text{ J} = K_{\text{rot}}(5/3)$, or $K_{\text{rot}} = 58.8\text{ J}$.

(b) The translational kinetic energy is $K_{\text{trans}} = (98-58.8)\text{J} = 39.2\text{ J}$.

85. (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. We obtain

$$I = (180\text{ kg})\,(0.910\text{ m})^2 = 149\text{ kg}\cdot\text{m}^2.$$

(b) An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, mvR, where R is the radius of the merry-go-round. Therefore,

$$\left|\vec{L}_{\text{child}}\right| = (44.0\text{ kg})(3.00\text{ m/s})(1.20\text{ m}) = 158\text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) No external torques act on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved. The initial angular momentum is given by mvR; the final angular momentum is given by $(I + mR^2)\,\omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = \left(I + mR^2\right)\omega$ and

$$\omega = \frac{mvR}{I+mR^2} = \frac{158\text{ kg}\cdot\text{m}^2/\text{s}}{149\text{ kg}\cdot\text{m}^2 + (44.0\text{ kg})(1.20\text{ m})^2} = 0.744\text{ rad/s}.$$

87. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,'\times\vec{v} = \left(x'v_y - y'v_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\text{i}}$ or the $-\hat{\text{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\text{i}}\times\hat{\text{i}} = 0$) that $\vec{\ell} = m\left(\vec{r}\,'\times\vec{v}\right) = 0$ in this case.

(b) The net force is in the $-\hat{\mathrm{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\hat{\mathrm{i}} + 5.0\hat{\mathrm{j}}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the +*x* or –*x* directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_{\mathrm{o}} \times \vec{v}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-2.0t^3\right)\Big)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = -30t^3\hat{\mathrm{k}}$ in SI units $\left(\mathrm{kg \cdot m^2/s}\right)$.

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{\mathrm{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_{\mathrm{o}} \times \vec{a}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-6.0t^2\right)\Big)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = -90t^2\hat{\mathrm{k}}$ in SI units $(\mathrm{N \cdot m})$.

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\hat{\mathrm{i}} - 5.0\hat{\mathrm{j}}$ (with SI units understood) and points from (2.0, –5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the +*x* or –*x* directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_{\mathrm{o}} \times \vec{v}\right) = -(3.0)\Big((2.0)(0) - (-5.0)\left(-2.0t^3\right)\Big)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = 30t^3\hat{\mathrm{k}}$ in SI units $\left(\mathrm{kg \cdot m^2/s}\right)$.

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{\mathrm{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_{\mathrm{o}} \times \vec{a}\right) = -(3.0)\Big((2.0)(0) - (-5.0)\left(-6.0t^2\right)\Big)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = 90t^2\hat{\mathrm{k}}$ in SI units $(\mathrm{N \cdot m})$.

95. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 11-3 (*pure rotation* about point P) but use torques instead of energy. The torque (relative to point P) is $\tau = I_P\alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the $F_{\text{app}} = 12\text{ N}$ force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P\alpha = \left(\frac{3}{2}MR^2\right)\alpha = 2RF_{\text{app}}$$

which leads to

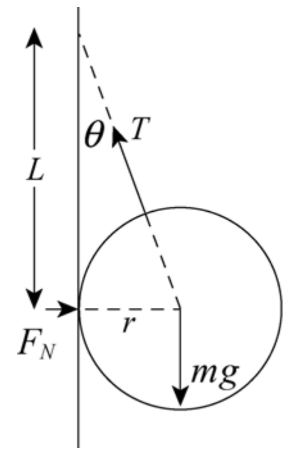
$$\alpha = \frac{2RF_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3MR} = \frac{4(12\text{ N})}{3(10\text{ kg})(0.10\text{ m})} = 16\,\text{rad/s}^2.$$

Hence, $a_{\text{com}} = R\alpha = 1.6\text{ m/s}^2$.

(b) As shown above, $\alpha = 16\text{ rad/s}^2$.

(c) Applying Newton's second law in its linear form yields $(12\text{ N}) - f = Ma_{\text{com}}$. Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N, i.e., $\vec{f} = (4.0\text{ N})\hat{\text{i}}$.

5. Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives



vertical component : $T\cos\theta - mg = 0$

horizontal component: $F_N - T\sin\theta = 0$.

(a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)\sqrt{(0.080\text{ m})^2+(0.042\text{ m})^2}}{0.080\text{ m}} = 9.4\text{ N}.$$

(b) We solve the second equation for the normal force: $F_N = T\sin\theta$.

Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)(0.042\text{ m})}{(0.080\text{ m})} = 4.4\text{ N}.$$

7. We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$
$$F_1 d + W(L-d) = 0$$

(a) The second equation gives

$$F_1 = -\frac{L-d}{d}W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N}$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N.

(b) Since F_1 is negative, indicating that this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N}$

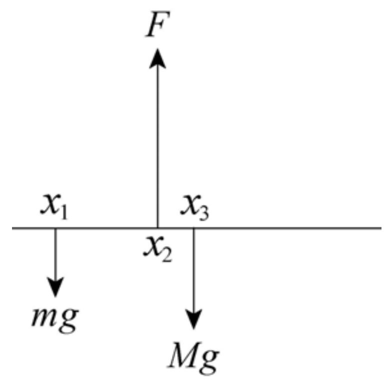
which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

11. The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram below. The nickels are at $x = x_1 = 0.120$ m, and m is their total mass. The knife edge is at $x = x_2 = 0.455$ m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500$ m. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:

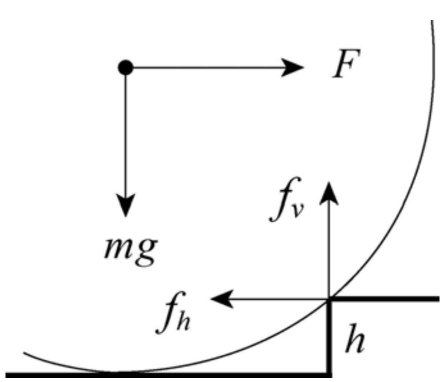


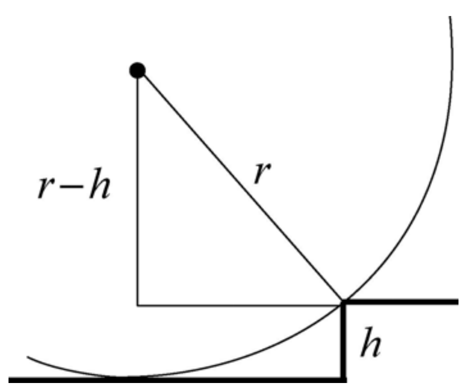
$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

Thus,

$$M = \frac{x_2 - x_1}{x_3 - x_2}\,m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}}\right)(10.0\text{g}) = 74.4\ \text{g}.$$

21. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.





We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r – h$, where r is the radius of the wheel and h is the height of the step.

The distance from the line of mg to the corner is $\sqrt{r^2+(r-h)^2} = \sqrt{2rh-h^2}$. Thus,

$$F(r-h) - mg\sqrt{2rh-h^2} = 0 .$$

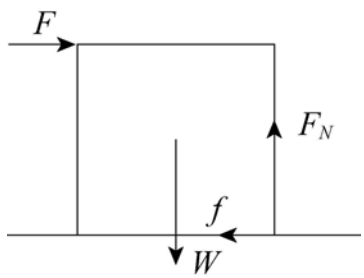
The solution for F is

$$F = \frac{\sqrt{2rh-h^2}}{r-h}mg = \frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m})-(3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m})-(3.00\times10^{-2}\,\text{m})}(0.800\text{ kg})(9.80\text{ m/s}^2)$$
$$= 13.6\text{ N}.$$

33. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram below. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F – f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL – WL/2 = 0.$$

Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

$$F = \frac{W}{2} = \frac{890\text{ N}}{2} = 445\text{ N}.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium $F_N = W = 890$ N and $f = F = 445$ N, so

$$\mu_s = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

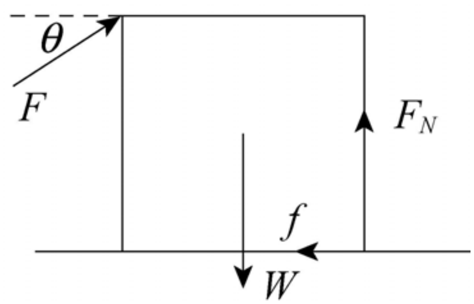
$$FL\cos\theta + FL\sin\theta - WL/2 = 0,$$

with the solution

$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45^\circ$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45^\circ} = \frac{890\,\text{N}}{4\cos 45^\circ} = 315\,\text{N}.$$



43. (a) The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross–sectional area of the rod. In this case F is the weight of the object hung on the end: $F = mg$, where m is the mass of the object. If r is the radius of the rod then $A = \pi r^2$. Thus, the shear stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5\times10^6\,\text{N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F/A}{\Delta x / L}$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F/A)L}{G} = \frac{(6.5\times10^{6}\ \text{N/m}^2)(0.053\,\text{m})}{3.0\times10^{10}\ \text{N/m}^2} = 1.1\times10^{-5}\ \text{m}.$$

55. (a) The forces acting on bucket are the force of gravity, down, and the tension force of cable A, up. Since the bucket is in equilibrium and its weight is

$$W_B = m_B g = (817\text{k}g)(9.80\,\text{m/s}^2) = 8.01\times10^{3}\,\text{N},$$

the tension force of cable A is $T_A = 8.01\times10^{3}\,\text{N}$.

(b) We use the coordinates axes defined in the diagram. Cable A makes an angle of θ_2 = 66.0º with the negative y axis, cable B makes an angle of 27.0º with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means T_B cos 27.0º – T_A cos 66.0º = 0 and

$$T_B = \frac{\cos 66.0°}{\cos 27.0°}T_A = \left(\frac{\cos 66.0°}{\cos 27.0°}\right)(8.01\times10^{3}\,\text{N}) = 3.65\times10^{3}\text{N}.$$

(c) The x components must also sum to zero. This means T_C + T_B sin 27.0º – T_A sin 66.0º = 0 and

$$\begin{aligned} T_C &= T_A \sin 66.0° - T_B \sin 27.0° = (8.01\times10^{3}\,\text{N})\sin 66.0° - (3.65\times10^{3}\,\text{N})\sin 27.0° \\ &= 5.66\times10^{3}\ \text{N}. \end{aligned}$$

61. We denote the mass of the slab as m, its density as ρ, and volume as $V = LTW$. The angle of inclination is $\theta = 26°$.

(a) The component of the weight of the slab along the incline is

$$\begin{aligned} F_1 &= mg\sin\theta = \rho Vg\sin\theta \\ &= (3.2\times10^{3}\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\sin 26° \approx 1.8\times10^{7}\ \text{N}. \end{aligned}$$

(b) The static force of friction is

$$\begin{aligned} f_s &= \mu_s F_N = \mu_s mg\cos\theta = \mu_s \rho Vg\cos\theta \\ &= (0.39)(3.2\times10^{3}\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\cos 26° \approx 1.4\times10^{7}\ \text{N}. \end{aligned}$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77\times10^{7}\,\text{N} - 1.42\times10^{7}\ \text{N} = 3.5\times10^{6}\ \text{N}.$$

If the minimum number of bolts needed is n, then $F_2/nA \le 3.6\times10^{8}\ \text{N/m}^2$, or

$$n \geq \frac{3.5\times10^6 \text{ N}}{(3.6\times10^8 \text{ N/m}^2)(6.4\times10^{-4} \text{ m}^2)} = 15.2$$

Thus 16 bolts are needed.

63. Analyzing forces at the knot (particularly helpful is a graphical view of the vector right–triangle with horizontal "side" equal to the static friction force f_s and vertical "side" equal to the weight m_Bg of block B), we find $f_s = m_Bg \tan\theta$ where $\theta = 30°$. For f_s to be at its maximum value, then it must equal $\mu_s m_A g$ where the weight of block A is m_Ag= (10 kg)(9.8 m/s^2). Therefore,

$$\mu_s m_A g = m_B g \tan\theta \Rightarrow \mu_s = \frac{5.0}{10}\tan 30° = 0.29.$$

65. With the pivot at the hinge, Eq. 12-9 leads to

$$-\,mg\sin\theta_1\frac{L}{2} + TL\sin(180° - \theta_1 - \theta_2) = 0 .$$

where $\theta_1 = 60°$ and $T = mg/2$. This yields $\theta_2 = 60°$.

81. When it is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point O where P is being applied. Thus, the line of action of $\vec{F}_N$ passes through point O and exerts no torque about O (of course, a similar observation applied to the pull P). Since $F_N = mg$ in this problem, we have $f_{smax} = \mu mg$ applied a distance h away from O. And the line of action of force of gravity (of magnitude mg), which is best pictured as a concentrated force at the center of the cube, is a distance $L/2$ away from O. Therefore, equilibrium of torques about O produces

$$\mu mgh = mg\left(\frac{L}{2}\right) \Rightarrow \mu = \frac{L}{2h} = \frac{(8.0 \text{ cm})}{2(7.0 \text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

(a) For it to slide but not tip, a value of μ *less* than that derived above is needed, since then — static friction will be exceeded for a smaller value of P, before the pull is strong enough to cause it to tip. Thus, $\mu < L/2h = 0.57$ is required.

(b) And for it to tip but not slide, we need μ *greater* than that derived above is needed, since now — static friction will not be exceeded even for the value of P which makes the cube rotate about its front lower corner. That is, we need to have $\mu > L/2h = 0.57$ in this case.

85. We choose an axis through the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0\text{ m})/5 = 1.6\,\text{m}$; in other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is $r_\perp = (4/5)(8.0\text{ m}) = 6.4\,\text{m}$. The moment arm for the weight is half the horizontal distance from the wall to the base of the ladder; this works out to be $\sqrt{(10\text{ m})^2 - (8\text{ m})^2}/2 = 3.0\,\text{m}$. Similarly, the moment arms for the x and y components of the force at the ground $\left(\vec{F}_g\right)$ are 8.0 m and 6.0 m, respectively. Thus, with lengths in meters, we have

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - F_{gy}(6.0\text{ m}) = 0.$$

In addition, from balancing the vertical forces we find that $W = F_{gy}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - W(6.0\text{ m}) = 0.$$

(a) With $F = 50$ N and $W = 200$ N, the above equation yields $F_{gx} = 35$ N. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(b) With $F = 150$ N and $W = 200$ N, the above equation yields $F_{gx} = -45$ N. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(c) Note that the phrase "start to move towards the wall" implies that the friction force is pointed away from the wall (in the $-\hat{\text{i}}$ direction). Now, if $f = -F_{gx}$ and $F_N = F_{gy} = 200$ N are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with $\mu_s = 0.38$, then we find $F_{gx} = -76$ N. Returning this to the above equation, we obtain

$$F = \frac{(200\,\text{N})(3.0\,\text{m}) + (76\,\text{N})(8.0\,\text{m})}{6.4\,\text{m}} = 1.9\times 10^2\text{ N}.$$

Chapter 13 – Student Solutions Manual

1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.2\,\mathrm{kg}\right)\left(2.4\,\mathrm{kg}\right)}{2.3\times10^{-12}\,\mathrm{N}}} = 19\,\mathrm{m}$$

7. At the point where the forces balance $GM_e m/r_1^2 = GM_s m/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{\left(d - r_1\right)^2}.$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s}+\sqrt{M_e}} = \frac{\left(150\times10^9\ \mathrm{m}\right)\sqrt{5.98\times10^{24}\ \mathrm{kg}}}{\sqrt{1.99\times10^{30}\ \mathrm{kg}}+\sqrt{5.98\times10^{24}\ \mathrm{kg}}} = 2.60\times10^8\ \mathrm{m}.$$

Values for M_e, M_s, and d can be found in Appendix C.

17. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(4.9\,\mathrm{m/s^2}\right)}} - 6.37\times10^6\,\mathrm{m} = 2.6\times10^6\,\mathrm{m}.$$

29. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \mathrm{km}}{3.45\times10^3\ \mathrm{km}}\right)^3 = 0.74.$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_g M = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2} a_{gE} = 0.11\left(\frac{0.65\times10^4 \text{ km}}{3.45\times10^3 \text{ km}}\right)^2 \left(9.8 \text{ m/s}^2\right) = 3.8 \text{ m/s}^2.$$

(c) If v is the escape speed, then, for a particle of mass m

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

For Mars, the escape speed is

$$v = \sqrt{\frac{2(6.67\times10^{-11} \text{ m}^3/\text{s}^2\cdot\text{kg})(0.11)\left(5.98\times10^{24} \text{ kg}\right)}{3.45\times10^6 \text{ m}}} = 5.0\times10^3 \text{ m/s}.$$

37. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0 \text{ m/s}^2)(500\times10^3 \text{ m})} = 1.7\times10^3 \text{ m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)}.$$

The solution for h is

$$h=\frac{2a_gR^2}{2a_gR-v^2}-R=\frac{2(3.0\text{ m/s}^2)(500\times10^3\text{ m})^2}{2(3.0\text{ m/s}^2)(500\times10^3\text{ m})-(1000\text{ m/s})^2}-(500\times10^3\text{ m})$$
$$=\ 2.5\times10^5\text{ m}.$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h}=-\frac{GMm}{R}+\frac{1}{2}mv^2.$$

We substitute a_gR^2 for GM and cancel m, obtaining

$$-\frac{a_gR^2}{R+h}=-a_gR+\frac{1}{2}v^2.$$

The solution for v is

$$v=\sqrt{2a_gR-\frac{2a_gR^2}{R+h}}=\sqrt{2(3.0\text{ m/s}^2)(500\times10^3\text{ m})-\frac{2(3.0\text{ m/s}^2)(500\times10^3\text{ m})^2}{(500\times10^3\text{ m})+(1000\times10^3\text{ m})}}$$
$$=1.4\times10^3\text{ m/s}.$$

39. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i}=-\frac{2GM^2}{r_i}+Mv^2.$$

The solution for v is

$$v=\sqrt{\frac{GM}{r_i}}=\sqrt{\frac{(6.67\times10^{-11}\text{ m}^3/\text{s}^2\cdot\text{kg})(10^{30}\text{ kg})}{10^{10}\text{ m}}}=8.2\times10^4\text{ m/s}.$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$v = \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} = \sqrt{(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})\left(\frac{1}{2\times10^5\ \text{m}} - \frac{1}{10^{10}\ \text{m}}\right)}$$
$$= 1.8\times10^7\ \text{m/s}.$$

45. Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N M$ and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2(2.2\times10^{20}\ \text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(7.88\times10^{15}\ \text{s})^2(2.0\times10^{30}\ \text{kg})} = 5.1\times10^{10}.$$

47. (a) The greatest distance between the satellite and Earth's center (the apogee distance) is $R_a = (6.37\times10^6\ \text{m} + 360\times10^3\ \text{m}) = 6.73\times10^6$ m. The least distance (perigee distance) is $R_p = (6.37\times10^6\ \text{m} + 180\times10^3\ \text{m}) = 6.55\times10^6$ m. Here 6.37×10^6 m is the radius of Earth. From Fig. 13-13, we see that the semi-major axis is

$$a = \frac{R_a + R_p}{2} = \frac{6.73\times10^6\ \text{m} + 6.55\times10^6\ \text{m}}{2} = 6.64\times10^6\ \text{m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73\times10^6\ \text{m} - 6.55\times10^6\ \text{m}}{6.73\times10^6\ \text{m} + 6.55\times10^6\ \text{m}} = 0.0136.$$

61. (a) We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_e = 2(150\times10^9\ \text{m}) = 300\times10^9$ m. Thus,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2(300\times10^9\ \text{m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(1.99\times10^{30}\text{kg})}} = 8.96\times10^7\ \text{s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T = 2.8$ y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e = (m/m_e)\,(r_e/r)$. We substitute $m = 2.0 \times 10^{-4} m_e$ and $r = 2r_e$ to obtain $K/K_e = 1.0 \times 10^{-4}$.

75. (a) Using Kepler's law of periods, we obtain

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right) r^3} = 2.15 \times 10^4 \text{ s} .$$

(b) The speed is constant (before she fires the thrusters), so $v_o = 2\pi r/T = 1.23 \times 10^4$ m/s.

(c) A two percent reduction in the previous value gives $v = 0.98v_o = 1.20 \times 10^4$ m/s.

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = 2.17 \times 10^{11}$ J.

(e) The potential energy is $U = -GmM/r = -4.53 \times 10^{11}$ J.

(f) Adding these two results gives $E = K + U = -2.35 \times 10^{11}$ J.

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = \frac{-GMm}{2E} = 4.04 \times 10^7 \text{ m} .$$

(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period is

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right) a^3} = 2.03 \times 10^4 \text{ s} .$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

(i) Elliptical orbit has a smaller period.

79. We use $F = Gm_s m_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15$ m + 3 m = 18 m. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(20\mathrm{kg}\right)\left(7.0\mathrm{kg}\right)}{\left(18\mathrm{m}\right)^2} = 2.9\times10^{-11}\,\mathrm{N}.$$

83. (a) We write the centripetal acceleration (which is the same for each, since they have identical mass) as $r\omega^2$ where ω is the unknown angular speed. Thus,

$$\frac{G(M)(M)}{(2r)^2} = \frac{GM^2}{4r^2} = Mr\omega^2$$

which gives $\omega = \frac{1}{2}\sqrt{MG/r^3} = 2.2\times10^{-7}$ rad/s.

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4\ \mathrm{m/s}\ .$$

87. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$ (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0 (-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8$ m/s^2 on the surface of Earth is given by $a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2$, where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\ \mathrm{m/s^2})(6.37\times10^6\ \mathrm{m})} = 7.9\times10^3\ \mathrm{m/s}\ .$$

93. The magnitude of the net gravitational force on one of the smaller stars (of mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}\left(M + \frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M + \frac{m}{4}\right) = m\frac{v^2}{r} \quad \text{where } \; v = \frac{2\mathrm{p}r}{T}.$$

Plugging in for speed *v*, we arrive at an equation for period *T*:

$$T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/4)}}.$$

Chapter 14 – Student Solutions Manual

1. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus

$$\Delta p = (42\ \text{N})/\pi(0.011\ \text{m})^2 = 1.1 \times 10^5\ \text{Pa}.$$

This is equivalent to 1.1 atm.

3. The air inside pushes outward with a force given by p_iA, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by p_oA, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since 1 atm = 1.013×10^5 Pa,

$$F = (1.0\ \text{atm} - 0.96\ \text{atm})(1.013\times10^5\ \text{Pa/atm})(3.4\ \text{m})(2.1\ \text{m}) = 2.9 \times 10^4\ \text{N}.$$

19. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is

$$W = \rho A(h_1 - h)g(h - h_2).$$

We substitute $h = (h_1 + h_2)/2$ to obtain

$$\begin{aligned} W &= \frac{1}{4}\rho g A\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\times10^{-4}\text{m}^2)(1.56\ \text{m} - 0.854\ \text{m})^2 \\ &= 0.635\ \text{J} \end{aligned}.$$

27. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly 1.3 kg/m^3. Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5\ \text{Pa}}{(1.3\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 7.9 \times 10^3\ \text{m} = 7.9\ \text{km}.$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g\ dy\ .$$

Assuming $\rho = \rho_0\,(1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8$ m/s^2 for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 gh.$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01\times 10^5\ \text{Pa})}{(1.3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 16\times 10^3\ \text{m} = 16\ \text{km}.$$

31. (a) The anchor is completely submerged in water of density ρ_w. Its effective weight is $W_{\text{eff}} = W - \rho_w gV$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 2.04\times 10^{-2}\ \text{m}^3.$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 14-1). Its weight in air is

$$W = mg = \rho Vg = \left(7870\ \text{kg/m}^3\right)(2.04\times 10^{-2}\ \text{m}^3)\left(9.80\ \text{m/s}^2\right) = 1.57\times 10^3\ \text{N}.$$

35. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain

$$\rho_b = 2\rho_w/3 = 2(1000\ \text{kg/m}^3)/3 \approx 6.7\times 10^2\ \text{kg/m}^3.$$

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 7.4\times 10^2\ \text{kg/m}^3$.

37. (a) The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)\left(800\ \text{kg/m}^3\right)(0.090\ \text{m})^3 = 1.22\ \text{kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090\ \text{m})^3 - (0.080\ \text{m})^3\right) = 9.09\times 10^{-4}\ \text{m}^3.$$

The density is

$$\rho_m = \frac{1.22 \text{ kg}}{9.09 \times 10^{-4} \text{ m}^3} = 1.3 \times 10^3 \text{ kg/m}^3 .$$

49. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 (NA_2) \quad \Rightarrow \quad v_2 = \frac{A_1}{NA_2} v_1 = \frac{R^2}{Nr^2} v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{Nd^2} v_1 = \frac{(1.9\,\text{cm})^2}{24(0.13\,\text{cm})^2}(0.91\,\text{m/s}) = 8.1\,\text{m/s}.$$

53. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by Δmgh, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta m v^2$, where v is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2}\Delta m v^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

Now the rate of mass flow is $\Delta m / \Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$ and

$$\rho_w A v = (1000 \text{ kg/m}^3)(3.14 \times 10^{-4} \text{ m}^2)(5.00 \text{ m/s}) = 1.57 \text{ kg/s}.$$

Thus,

$$P = \rho A v\left(gh + \frac{1}{2}v^2\right) = (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\,\text{W}.$$

55. (a) We use the equation of continuity: $A_1 v_1 = A_2 v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus

$$v_2 = (A_1/A_2)v_1 = [(4.0 \text{ cm}^2)/(8.0 \text{ cm}^2)]\,(5.0 \text{ m/s}) = 2.5 \text{m/s}.$$

(b) We use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \,,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left[(5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right] + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa.} \end{aligned}$$

59. (a) We use the Bernoulli equation: $p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \tfrac{1}{2}\rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g\left(h_1 - h_2\right)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.30\,\text{m}\right)} = 2.42\,\text{m/s}.$$

The flow rate is $A_2 v_2 = (6.5 \times 10^{-4}\text{ m}^2)(2.42\text{ m/s}) = 1.6 \times 10^{-3}\text{ m}^3\text{/s}$.

(b) We use the equation of continuity: $A_2 v_2 = A_3 v_3$, where $A_3 = \tfrac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\text{ m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\tfrac{1}{2}\rho v_2^2 + \rho g h_2 = \tfrac{1}{2}\rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{\left(4.84\,\text{m/s}\right)^2 - \left(2.42\,\text{m/s}\right)^2}{2\left(9.8\,\text{m/s}^2\right)} = 0.90\text{ m.}$$

67. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p + \tfrac{1}{2}\rho v^2 = \tfrac{1}{2}\rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p + \tfrac{1}{2}\rho v^2 = \tfrac{1}{2}\rho\left(A/a\right)^2 v^2$. We solve for v. The result is

$$v = \sqrt{\frac{2\Delta p}{\rho\left((A/a)^2 - 1\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(A^2 - a^2\right)}}.$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32\times10^{-4}\,\mathrm{m}^2)^2(55\times10^3\,\mathrm{Pa} - 41\times10^3\,\mathrm{Pa})}{(1000\,\mathrm{kg/m}^3)\left((64\times10^{-4}\,\mathrm{m}^2)^2 - (32\times10^{-4}\,\mathrm{m}^2)^2\right)}} = 3.06\,\mathrm{m/s}.$$

Consequently, the flow rate is

$$Av = (64\times10^{-4}\,\mathrm{m}^2)(3.06\ \mathrm{m/s}) = 2.0\times10^{-2}\,\mathrm{m}^3/\mathrm{s}.$$

75. If we examine both sides of the U-tube at the level where the low-density liquid (with ρ = 0.800 g/cm^3 = 800 kg/m^3) meets the water (with ρ_w = 0.998 g/cm^3 = 998 kg/m^3), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where h = 8.00 cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is h_w = (800/998)(8.00 cm) = 6.41 cm. The volume of water in that column is therefore $\pi r^2 h_w = \pi(1.50\ \mathrm{cm})^2(6.41\ \mathrm{cm}) = 45.3\ \mathrm{cm}^3$.

3. (a) The amplitude is half the range of the displacement, or $x_m = 1.0$ mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi\left(120 \text{ Hz}\right)\left(1.0\times10^{-3} \text{ m}\right) = 0.75 \text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = \left(2\pi f\right)^2 x_m = \left(2\pi\left(120 \text{ Hz}\right)\right)^2\left(1.0\times10^{-3} \text{ m}\right) = 5.7\times10^2 \text{ m/s}^2.$$

7. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500 \text{ s}) = 2.00$ Hz.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00 \text{ Hz}) = 12.6$ rad/s.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) Let x_m be the amplitude. The maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6$ N.

9. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g / x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{1.0\times10^{-6} \text{m}}} = 498 \text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

17. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\ \text{m/s}^2)}{(2\pi \times 2.0\ \text{Hz})^2} = 0.031\ \text{m}.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

19. (a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50$ s,

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\ \text{s}}{1.5\ \text{s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\ \text{s}}{1.5\ \text{s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{dx_2}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for t = 0.50 s and find they are both negative-valued, indicating that the particles are moving in the same direction.

27. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2 \text{ N/m}\right)\left(0.024 \text{ m}\right)^2 = 3.7\times10^{-2} \text{ J}.$$

29. The total energy is given by $E = \frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

(a) The fraction of the energy that is kinetic is

$$\frac{K}{E} = \frac{E-U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

where the result from part (b) has been used.

(b) When $x = \frac{1}{2}x_m$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kx_m^2$. The ratio is

$$\frac{U}{E} = \frac{kx_m^2/8}{kx_m^2/2} = \frac{1}{4} = 0.25.$$

(c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve x^2/x_m^2 = 1/2 for x. We should get $x = x_m/\sqrt{2}$.

39. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m\sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad/s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta.$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

43. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2 + \frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned}
I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\
&= \frac{1}{2}(0.500\,\text{kg})(0.100\,\text{m})^2 + (0.500\,\text{kg})(0.500\,\text{m} + 0.100\,\text{m})^2 + \frac{1}{3}(0.270\,\text{kg})(0.500\,\text{m})^2 \\
&= 0.205\,\text{kg}\cdot\text{m}^2.
\end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\text{ m} + 0.100\text{ m} = 0.600\text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\text{ m})/2 = 0.250\text{ m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\text{ kg})(0.600\text{ m}) + (0.270\text{ kg})(0.250\text{ m})}{0.500\text{ kg} + 0.270\text{ kg}} = 0.477\text{ m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\text{ kg}\cdot\text{m}^2}{(0.500\text{ kg} + 0.270\text{ kg})(9.80\text{ m/s}^2)(0.447\text{ m})}} = 1.50\text{ s}.$$

51. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = 2\pi/\omega = 2\pi\sqrt{I/C},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1 - \cos\theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}\ .$$

If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 10-2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With m = 0.600 kg and k = 1850 N/m, we obtain T = 0.0653 s.

57. (a) We want to solve $e^{-bt/2m}$ = 1/3 for t. We take the natural logarithm of both sides to obtain $-bt/2m$ = ln(1/3). Therefore, $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$. Thus,

$$t = \frac{2(1.50\ \text{kg})}{0.230\ \text{kg/s}}\ln 3 = 14.3\ \text{s}.$$

(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\,\text{N/m}}{1.50\,\text{kg}} - \frac{(0.230\,\text{kg/s})^2}{4(1.50\,\text{kg})^2}} = 2.31\,\text{rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\ \text{rad/s}) = 2.72$ s and the number of oscillations is

$$t/T = (14.3\ \text{s})/(2.72\ \text{s}) = 5.27.$$

75. (a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives

$$f = (1/2\pi)\sqrt{(9.80\ \text{m/s}^2)/(2.0\ \text{m})} = 0.35\ \text{Hz}.$$

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g} - \vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2}{2.0\,\text{m}}} = 0.39\ \text{Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

83. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is 180/(60 s) = 3.0 Hz and the amplitude is half the stroke, or x_m = 0.38 m. Thus,

$$v_m = 2\pi(3.0\ \text{Hz})(0.38\ \text{m}) = 7.2\ \text{m/s}.$$

89. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where y = 0.096 m is the elongation of the spring at equilibrium, k is the spring constant, and m = 1.3 kg is the mass of the block. Thus

$$k = mg/y = (1.3\ \text{kg})(9.8\ \text{m/s}^2)/(0.096\ \text{m}) = 1.33\times10^2\ \text{N/m}.$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3\ \text{kg}}{133\ \text{N/m}}} = 0.62\ \text{s}.$$

(c) The frequency is $f = 1/T$ = 1/0.62 s = 1.6 Hz.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3\ \text{kg})(9.8\ \text{m/s}^2)(0.146\ \text{m}) + \frac{1}{2}(133\ \text{N/m})(0.146\ \text{m})^2 = -0.44\ \text{J}.$$

When the block is at the equilibrium point, the elongation of the spring is y = 9.6 cm and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3\ \text{kg})(9.8\ \text{m/s}^2)(0.096\ \text{m}) + \frac{1}{2}(133\ \text{N/m})(0.096\ \text{m})^2 = -0.61\ \text{J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\ \text{J} + 0.61\ \text{J})}{1.3\ \text{kg}}} = 0.51\ \text{m/s}.$$

91. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given v = 5.2 m/s at x = 0 is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20\ \text{rad/s}$).

(a) Since $\omega = 2\pi f$, we find $f = 3.2$ Hz.

(b) We have $v_m = 5.2$ m/s = (20 rad/s)x_m, which leads to $x_m = 0.26$ m.

(c) With meters, seconds and radians understood,

$$x = (0.26 \text{ m})\cos(20t + \phi)$$
$$v = -(5.2 \text{ m/s})\sin(20t + \phi).$$

The requirement that $x = 0$ at $t = 0$ implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when $t = 0$; that choice is $\phi = -\pi/2$. Therefore,

$$x = 0.26\cos\left(20t - \frac{\pi}{2}\right) = 0.26\sin(20t).$$

15. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope:

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m}.$$

Thus,

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

17. (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100 \text{ Hz}) = 628 \text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

21. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses y = 0 at about x = 15 cm and again with the same slope at about x = 55 cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25\times 10^{-3}\,\text{kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda$ = (12 m/s)/(0.40 m) = 30 Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})\,(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9\times 10^2$ rad/s

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin\phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}$. The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x,t) = \left(5.0\times 10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

31. The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) = 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

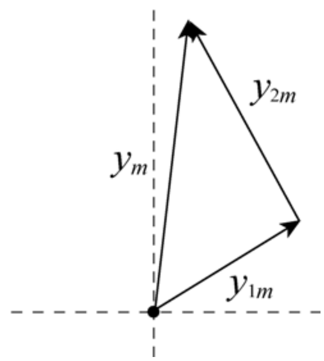
where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41 y_m .$$

35. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus $y_m = 5.0$ cm.



41. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\text{ N}}{(10.0\text{ m})\,(0.100\text{ kg})}} = n\ (7.91\text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\text{ Hz}) = 23.7$ Hz.

43. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\text{ N})\,(8.40\text{ m})}{0.120\text{ kg}}} = 82.0\text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\text{ m}) = 16.8$ m.

(c) The frequency is $f = v/\lambda = (82.0\text{ m/s})/(16.8\text{ m}) = 4.88$ Hz.

47. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated

with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\text{ Hz}}{420\text{ Hz} - 315\text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\text{ Hz})/3 = 105\text{ Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then

$$v = \lambda f = 2Lf = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

53. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The amplitude y_m is half the maximum displacement of the standing wave, or 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L = 3.0$m, $\lambda = 2.0$ m. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0\text{ m}) = 3.1\text{ m}^{-1}$.

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50\text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50\text{ Hz}) = 314$ rad/s.

(d) The two waves are

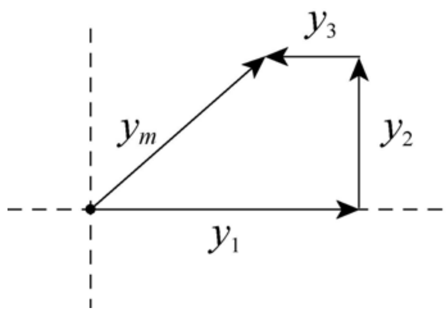
$$y_1 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

61. (a) The phasor diagram is shown here: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave.



The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6} y_1 = 0.83 y_1.$$

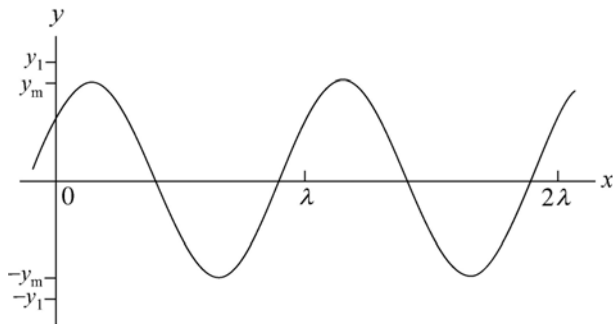
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6} y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



69. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

77. (a) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120\,\text{N}}{8.70 \times 10^{-3}\,\text{kg}/1.50\,\text{m}}} = 144 \text{ m/s}.$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50 \text{ m}) = 3.00$ m.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50$ m.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144 \text{ m/s})/(3.00 \text{ m}) = 48.0$ Hz.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144 \text{ m/s})/(1.50 \text{ m}) = 96.0$ Hz.

78. We use $P = \frac{1}{2}\mu v \omega^2 y_m^2 \propto v f^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1.$

(b) If the frequency is halved, then $P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1.$

87. (a) From the frequency information, we find $\omega = 2\pi f = 10\pi$ rad/s. A point on the rope undergoing simple harmonic motion (discussed in Chapter 15) has maximum speed as it passes through its "middle" point, which is equal to $y_m\omega$. Thus,

$$5.0 \text{ m/s} = y_m\omega \quad\Rightarrow\quad y_m = 0.16 \text{ m} \quad .$$

(b) Because of the oscillation being in the *fundamental* mode (as illustrated in Fig. 16-23(a) in the textbook), we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \quad\Rightarrow\quad \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2 \text{ N}.$$

(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$, and we observe that the anti-node having zero displacement at $t = 0$ suggests the use of sine instead of cosine for the simple harmonic motion factor. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m}) \sin[(1.57 \text{ m}^{-1})x] \sin[(31.4 \text{ rad/s})t]$$

89. (a) The wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}} = \sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}.$$

(b) The time required is

$$t = \frac{2\pi(\ell+\Delta\ell)}{v} = \frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k} = \text{const}$.

5. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives $d = \frac{1}{2} g t_f^2$, or $t_f = \sqrt{2d/g}$. The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d. Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

$$gd^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for d. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.

7. If d is the distance from the location of the earthquake to the seismograph and v_s is the speed of the S waves then the time for these waves to reach the seismograph is $t_s. = d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

so

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(3.0\,\text{min})(60\,\text{s/min})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9\times10^3\ \text{km}.$$

We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

9. (a) Using $\lambda = v/f$, where v is the speed of sound in air and f is the frequency, we find

$$\lambda = \frac{343\,\text{m/s}}{4.50\times10^6\ \text{Hz}} = 7.62\times10^{-5}\ \text{m}.$$

(b) Now, $\lambda = v/f$, where v is the speed of sound in tissue. The frequency is the same for air and tissue. Thus

$$\lambda = (1500 \text{ m/s})/(4.50 \times 10^6 \text{ Hz}) = 3.33 \times 10^{-4} \text{ m}.$$

19. Let L_1 be the distance from the closer speaker to the listener. The distance from the other speaker to the listener is $L_2 = \sqrt{L_1^2 + d^2}$, where d is the distance between the speakers. The phase difference at the listener is $\phi = 2\pi(L_2 - L_1)/\lambda$, where λ is the wavelength.

For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where n is an integer. Thus $\lambda = 2(L_2 - L_1)/(2n + 1)$. The frequency is

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{2\left(\sqrt{L_1^2 + d^2} - L_1\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range. This means n ranges from 0 to 28.

(a) The lowest frequency that gives minimum signal is (n = 0) $f_{\text{min},1} = 343$ Hz.

(b) The second lowest frequency is (n = 1) $f_{\text{min},2} = [2(1)+1]343 \text{ Hz} = 1029 \text{ Hz} = 3 f_{\text{min},1}$. Thus, the factor is 3.

(c) The third lowest frequency is (n=2) $f_{\text{min},3} = [2(2)+1]343 \text{ Hz} = 1715 \text{ Hz} = 5 f_{\text{min},1}$. Thus, the factor is 5.

For a maximum in intensity at the listener, $\phi = 2n\pi$, where n is any positive integer. Thus $\lambda = (1/n)\left(\sqrt{L_1^2 + d^2} - L_1\right)$ and

$$f = \frac{v}{\lambda} = \frac{nv}{\sqrt{L_1^2 + d^2} - L_1} = \frac{n(343\,\text{m/s})}{\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, n must be in the range from 1 to 29 for the frequency to be audible.

(d) The lowest frequency that gives maximum signal is (n =1) $f_{\text{max},1} = 686$ Hz.

(e) The second lowest frequency is (n = 2) $f_{\text{max},2} = 2(686 \text{ Hz}) = 1372 \text{ Hz} = 2 f_{\text{max},1}$. Thus, the factor is 2.

(f) The third lowest frequency is (n = 3) $f_{\max,3} = 3(686 \text{ Hz}) = 2058 \text{ Hz} = 3 f_{\max,1}$. Thus, the factor is 3.

25. The intensity is the rate of energy flow per unit area perpendicular to the flow. The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where A (= $4\pi r^2$) is the surface area of a sphere of radius r. Thus

$$P = 4\pi(2.50 \text{ m})^2 (1.91 \times 10^{-4} \text{ W/m}^2) = 1.50 \times 10^{-2} \text{ W}.$$

29. (a) Let I_1 be the original intensity and I_2 be the final intensity. The original sound level is β_1 = (10 dB) $\log(I_1/I_0)$ and the final sound level is β_2 = (10 dB) $\log(I_2/I_0)$, where I_0 is the reference intensity. Since $\beta_2 = \beta_1$ + 30 dB which yields

$$(10 \text{ dB}) \log(I_2/I_0) = (10 \text{ dB}) \log(I_1/I_0) + 30 \text{ dB},$$

or

$$(10 \text{ dB}) \log(I_2/I_0) - (10 \text{ dB}) \log(I_1/I_0) = 30 \text{ dB}.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 3$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of 1.0×10^3.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} = 32$.

43. (a) When the string (fixed at both ends) is vibrating at its lowest resonant frequency, exactly one-half of a wavelength fits between the ends. Thus, $\lambda = 2L$. We obtain

$$v = f\lambda = 2Lf = 2(0.220 \text{ m})(920 \text{ Hz}) = 405 \text{ m/s}.$$

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. If M is the mass of the (uniform) string, then $\mu = M/L$. Thus

$$\tau = \mu v^2 = (M/L)v^2 = [(800 \times 10^{-6} \text{ kg})/(0.220 \text{ m})] (405 \text{ m/s})^2 = 596 \text{ N}.$$

(c) The wavelength is $\lambda = 2L = 2(0.220 \text{ m}) = 0.440$ m.

(d) The frequency of the sound wave in air is the same as the frequency of oscillation of the string. The wavelength is different because the wave speed is different. If v_a is the speed of sound in air the wavelength in air is

$$\lambda_a = v_a/f = (343 \text{ m/s})/(920 \text{ Hz}) = 0.373 \text{ m}.$$

45. (a) Since the pipe is open at both ends there are displacement antinodes at both ends and an integer number of half-wavelengths fit into the length of the pipe. If L is the pipe length and λ is the wavelength then $\lambda = 2L/n$, where n is an integer. If v is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now $L = 0.457$ m, so

$$f = n(344 \text{ m/s})/2(0.457 \text{ m}) = 376.4n \text{ Hz}.$$

To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set $f = 1000$ Hz and solve for n, then set $f = 2000$ Hz and again solve for n. The results are 2.66 and 5.32, which imply that $n = 3$, 4, and 5 are the appropriate values of n. Thus, there are 3 frequencies.

(b) The lowest frequency at which resonance occurs is ($n = 3$) $f = 3(376.4 \text{ Hz}) = 1129$ Hz.

(c) The second lowest frequency at which resonance occurs is ($n = 4$)

$$f = 4(376.4 \text{ Hz}) = 1506 \text{ Hz}.$$

47. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f = (n/2L)\sqrt{\tau/\mu}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_2 = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_2 - f_1 = (1/2L)\sqrt{\tau/\mu}$ and

$$\tau = 4L^2\mu(f_2 - f_1)^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\text{ kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 = 45.3\,\text{N}.$$

53. Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is $f = v/\lambda = (1/2L)\sqrt{\tau/\mu}$, where $v = \sqrt{\tau/\mu}$ is the wave speed for the wire, τ is the tension in the wire, and μ is the linear mass density of the wire. Suppose the tension in one wire is τ and the oscillation frequency of that wire is f_1. The tension in the other wire is $\tau + \Delta\tau$ and its frequency is f_2. You want to calculate

$\Delta\tau/\tau$ for f_1 = 600 Hz and f_2 = 606 Hz. Now, $f_1 = (1/2L)\sqrt{\tau/\mu}$ and $f_2 = (1/2L)\sqrt{(\tau + \Delta\tau/\mu}$, so

$$f_2 / f_1 = \sqrt{(\tau + \Delta\tau)/\tau} = \sqrt{1 + (\Delta\tau/\tau)}.$$

This leads to $\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [(606\,\text{Hz})/(600\,\text{Hz})]^2 - 1 = 0.020.$

65. (a) The expression for the Doppler shifted frequency is

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector (the uncle), and v_S is the speed of the source (the locomotive). All speeds are relative to the air. The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is v_S = 10 m/s. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} + 10.00\,\text{m/s}}\right) = 485.8\,\text{Hz}.$$

(b) The girl is now the detector. Relative to the air she is moving with speed v_D = 10.00 m/s toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at v_S = 10.00 m/s away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f$ = 500.0 Hz.

(c) Relative to the air the locomotive is moving at v_S = 20.00 m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at v_D = 10.00 m/s toward the locomotive. Use the plus sign in the numerator. Thus

$$f' = f\,\frac{v + v_D}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}.$$

(d) Relative to the air the locomotive is moving at v_S = 20.00 m/s away from the girl and the girl is moving at v_D = 20.00 m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f$ = 500.0 Hz.

77. The siren is between you and the cliff, moving away from you and towards the cliff. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17–47 (and see the discussion in the textbook immediately after that equation regarding the selection of ± signs). The source is the siren with v_S = 10 m/s. The problem asks us to use v = 330 m/s for the speed of sound.

(a) With $f = 1000$ Hz, the frequency f_y you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = 970.6\,\text{Hz} \approx 9.7\times10^2\ \text{Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = 1031.3\,\text{Hz} \approx 1.0\times10^3\ \text{Hz}.$$

(c) The beat frequency is $f_c - f_y = 60$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

81. (a) With $r = 10$ m in Eq. 17–28, we have

$$I = \frac{P}{4\pi r^2} \quad\Rightarrow\quad P = 10\,\text{W}.$$

(b) Using that value of P in Eq. 17–28 with a new value for r, we obtain

$$I = \frac{P}{4\pi(5.0)^2} = 0.032\,\frac{\text{W}}{\text{m}^2}.$$

Alternatively, a ratio $I'/I = (r/r')^2$ could have been used.

(c) Using Eq. 17–29 with $I = 0.0080$ W/m^2, we have

$$\beta = 10\log\frac{I}{I_0} = 99\,\text{dB}$$

where $I_0 = 1.0\times10^{-12}$ W/m^2.

85. (a) The intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where ρ is the density of the medium, v is the speed of sound, ω is the angular frequency, and s_m is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v\omega s_m$, so $s_m = \Delta p_m/\rho v\omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w},$$

where the subscript a denotes air and the subscript w denotes water. Since $I_a = I_w$,

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})}{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}} = 59.7.$$

The speeds of sound are given in Table 17-1 and the densities are given in Table 15-1.

(b) Now, $\Delta p_{mw} = \Delta p_{ma}$, so

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})} = 2.81\times10^{-4}.$$

87. (a) When the right side of the instrument is pulled out a distance d the path length for sound waves increases by $2d$. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where λ is the wavelength. Thus $\lambda = 4d$ and, if v is the speed of sound, the frequency is

$$f = v/\lambda = v/4d = (343\ \text{m/s})/4(0.0165\ \text{m}) = 5.2\times10^3\ \text{Hz}.$$

(b) The displacement amplitude is proportional to the square root of the intensity (see Eq. 17–27). Write $\sqrt{I} = Cs_m$, where I is the intensity, s_m is the displacement amplitude, and C is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$. Solve

$$\sqrt{100} = C(s_{SAD} - s_{SBD}) \quad \text{and} \quad \sqrt{900} = C(s_{SAD} - s_{SBD})$$

for s_{SAD} and s_{SBD}. Add the equations to obtain

$$s_{SAD} = (\sqrt{100} + \sqrt{900} / 2C = 20/C,$$

then subtract them to obtain

$$s_{SBD} = (\sqrt{900} - \sqrt{100})/2C = 10/C.$$

The ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

101. (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right) \qquad \text{where } v_x = v_{\text{blood}} \cos\theta.$$

If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90\,\text{m/s}$$

where v = 1540 m/s, θ = 20°, and $R = 1 + 5495/5 \times 10^6$.

(c) We interpret the question as asking how Δf (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle θ changes. Since larger θ means smaller horizontal component of velocity v_x then we expect Δf to decrease towards zero as θ is increased towards 90°.

8. The change in length for the aluminum pole is

$$\Delta\ell = \ell_0 \alpha_{Al} \Delta T = (33\,\mathrm{m})(23\times10^{-6}\,/\,\mathrm{C°})(15\ \mathrm{°C}) = 0.011\,\mathrm{m}.$$

15. If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c \Delta T$. See Eq. 18-11. If β is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is $\Delta V_g = \beta V_c \Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\begin{aligned}\Delta V_g - \Delta V_c &= \left(\beta - 3\alpha_a\right) V_c \Delta T = \left[\left(5.1\times10^{-4}\,/\,\mathrm{C°}\right) - 3\left(23\times10^{-6}\,/\,\mathrm{C°}\right)\right]\left(100\,\mathrm{cm}^3\right)\left(6.0\,\mathrm{°C}\right)\\ &= 0.26\,\mathrm{cm}^3.\end{aligned}$$

21. Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha\ell_0\Delta T$. The old position of the half-bar, its new position, and the distance x that one end is displaced form a right triangle, with a hypotenuse of length ℓ, one side of length ℓ_0, and the other side of length x. The Pythagorean theorem yields $x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha\Delta T)^2 - \ell_0^2.$ Since the change in length is small we may approximate $(1 + \alpha\,\Delta T)^2$ by $1 + 2\alpha\,\Delta T$, where the small term $(\alpha\,\Delta T)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\,\Delta T - \ell_0^2 = 2\ell_0^2\alpha\,\Delta T$$

and

$$x = \ell_0\sqrt{2\alpha\,\Delta T} = \frac{3.77\,\mathrm{m}}{2}\sqrt{2\left(25\times10^{-6}/\mathrm{C°}\right)\left(32°\,\mathrm{C}\right)} = 7.5\times10^{-2}\ \mathrm{m}.$$

25. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236\,\mathrm{J/kg\cdot K})(0.130\,\mathrm{kg})(1235\mathrm{°C} - 288\mathrm{°C}) = 2.91\times10^4\ \mathrm{J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q = mL_F = \left(0.130\,\mathrm{kg}\right)\left(105\times10^3\ \mathrm{J/kg}\right) = 1.36\times10^4\ \mathrm{J}.$$

The total heat required is $(2.91\times10^4\ \mathrm{J} + 1.36\times10^4\ \mathrm{J}) = 4.27\times10^4\ \mathrm{J}$.

27. The mass m = 0.100 kg of water, with specific heat c = 4190 J/kg·K, is raised from an initial temperature T_i = 23°C to its boiling point T_f = 100°C. The heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater P multiplied by the time t; $Q = Pt$. Thus,

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(100°\,\text{C} - 23°\text{C})}{200\,\text{J/s}} = 160\,\text{s}.$$

41. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

• None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

• The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

• All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, suppose that no ice melts. The temperature of the water decreases from T_{Wi} = 25°C to some final temperature T_f and the temperature of the ice increases from T_{Ii} = –15°C to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W (T_{Wi} - T_f).$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\text{J / kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})} \\ &= 16.6°\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at T_f = 0°C, with mass m (< m_I) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + mL_F,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F.$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25°\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15°\mathrm{C})}{333\times10^3\,\mathrm{J/kg}} \\ &= 5.3\times10^{-2}\,\mathrm{kg} = 53\,\mathrm{g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain T_f = 2.5°C.

43. Over a cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$. Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write

$$p = \frac{10}{3} \text{ Pa} + \left(\frac{20}{3} \text{ Pa/m}^3\right)V,$$

where the coefficients were chosen so that p = 10 Pa when V = 1.0 m^3 and p = 30 Pa when V = 4.0 m^3. The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p\,dV = \int_1^4 \left(\frac{10}{3} + \frac{20}{3}V\right) dV = \left(\frac{10}{3}V + \frac{10}{3}V^2\right)\Bigg|_1^4 \\ &= \left(\frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3}\right) \text{J} = 60 \text{ J}. \end{aligned}$$

The BC portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p\Delta V = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}.$$

The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is $W = W_{AB} + W_{BC} + W_{CA}$ = 60 J – 90 J + 0 = –30 J and the total heat absorbed is $Q = W$ = –30 J. This means the gas loses 30 J of energy in the form of heat.

49. (a) The change in internal energy ΔE_{int} is the same for path *iaf* and path *ibf*. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along *iaf*

$$\Delta E_{\text{int}} = Q - W = 50 \text{ cal} - 20 \text{ cal} = 30 \text{ cal}.$$

Along *ibf* ,

$$W = Q - \Delta E_{\text{int}} = 36 \text{ cal} - 30 \text{ cal} = 6.0 \text{ cal}.$$

(b) Since the curved path is traversed from f to i the change in internal energy is –30 cal and $Q = \Delta E_{\text{int}} + W$ = –30 cal – 13 cal = – 43 cal.

(c) Let $\Delta E_{\text{int}} = E_{\text{int},\,f} - E_{\text{int},\,i}$. Then, $E_{\text{int},\,f} = \Delta E_{\text{int}} + E_{\text{int},\,i}$ = 30 cal + 10 cal = 40 cal.

(d) The work W_{bf} for the path bf is zero, so $Q_{bf} = E_{\text{int},\,f} - E_{\text{int},\,b}$ = 40 cal – 22 cal = 18 cal.

(e) For the path ibf, Q = 36 cal so $Q_{ib} = Q - Q_{bf}$ = 36 cal – 18 cal = 18 cal.

51. The rate of heat flow is given by

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L},$$

where k is the thermal conductivity of copper (401 W/m·K), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C. Thus,

$$P_{\text{cond}} = \frac{(401\,\text{W/m}\cdot\text{K})(90.0\times10^{-4}\,\text{m}^2)(125°\text{C} - 10.0°\text{C})}{0.250\,\text{m}} = 1.66\times10^3\ \text{J/s}.$$

The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

65. Let h be the thickness of the slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{h}$$

where k is the thermal conductivity of ice, T_H is the temperature of the water (0°C), and T_C is the temperature of the air above the ice (–10°C). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F\,\frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho Ah$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F\,\rho A\frac{dh}{dt}.$$

We equate the two expressions for P_{cond} and solve for dh/dt:

$$\frac{dh}{dt} = \frac{k\left(T_H - T_C\right)}{L_F \rho h}.$$

Since 1 cal = 4.186 J and 1 cm = 1 × 10^{-2} m, the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s·cm·K})\,(4.186 \text{ J/cal})/(1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m·K}.$$

The density of ice is ρ = 0.92 g/cm^3 = 0.92 × 10^3 kg/m^3. Thus,

$$\frac{dh}{dt} = \frac{\left(1.674\,\text{W/m}\cdot\text{K}\right)\left(0°\text{C} + 10°\text{C}\right)}{\left(333\times10^3\ \text{J/kg}\right)\left(0.92\times10^3\ \text{kg/m}^3\right)\left(0.050\,\text{m}\right)} = 1.1\times10^{-6}\ \text{m/s} = 0.40\,\text{cm/h}.$$

73. The work (the "area under the curve") for process 1 is $4p_iV_i$, so that

$$U_b - U_a = Q_1 - W_1 = 6p_iV_i$$

by the First Law of Thermodynamics.

(a) Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$). With $W_2 = 4p_iV_i + p_iV_i = 5p_iV_i$, we obtain

$$Q_2 = W_2 + U_b - U_a = 5p_iV_i + 6p_iV_i = 11p_iV_i.$$

(b) Path 3 starts at a and ends at b so that $\Delta U = U_b - U_a = 6p_iV_i$.

75. The volume of the disk (thought of as a short cylinder) is $\pi r^2 L$ where L = 0.50 cm is its thickness and r = 8.0 cm is its radius. Eq. 18-10, Eq. 18-11 and Table 18-2 (which gives $\alpha = 3.2 \times 10^{-6}$/C°) lead to

$$\Delta V = (\pi r^2 L)(3\alpha)(60°\text{C} - 10°\text{C}) = 4.83 \times 10^{-2}\ \text{cm}^3.$$

77. We have $W = \int p\, dV$ (Eq. 18-24). Therefore,

$$W = a\int V^2 dV = \frac{a}{3}\left(V_f^3 - V_i^3\right) = 23\ \text{J}.$$

81. Following the method of Sample Problem 18-4 (particularly its third Key Idea), we have

$$(900\,\tfrac{\text{J}}{\text{kg·C°}})(2.50\ \text{kg})(T_f - 92.0°\text{C}) + (4190\,\tfrac{\text{J}}{\text{kg·C°}})(8.00\ \text{kg})(T_f - 5.0°\text{C}) = 0$$

where Table 18-3 has been used. Thus we find T_f = 10.5°C.

82. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$. In this case $\Delta E_{\text{int}} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\,\text{kJ}.$$

83. This is similar to Sample Problem 18-3. An important difference with part (b) of that sample problem is that, in this case, the final state of the H_2O is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process:

$$Q = m\,(\,c_{\text{ice}}\,(0\ \text{C°} - \ (-150\ \text{C°})) \ + \ L_F \ \ + \ \ c_{\text{liquid}}\,(\,T_f \ - 0\ \text{C°}))$$

Thus,

$$T_f \ = \ \frac{Q/m - (c_{\text{ice}}(150°) + L_F\,)}{c_{\text{liquid}}} \ = \ 79.5\text{°C}\ .$$

7. (a) In solving $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T = (40.0 + 273.15)\,\text{K} = 313.15\ \text{K}$. And we convert the volume to SI units: 1000 cm^3 = 1000×10^{-6} m^3. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^5\ \text{Pa}\right)\left(1000 \times 10^{-6}\ \text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(313.15\,\text{K}\right)} = 3.88 \times 10^{-2}\ \text{mol}.$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{\left(1.06 \times 10^5\ \text{Pa}\right)\left(1500 \times 10^{-6}\ \text{m}^3\right)}{\left(3.88 \times 10^{-2}\ \text{mol}\right)\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 493\,\text{K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_iV_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is 1.03×10^5 Pa,

$$p_i = 1.03 \times 10^5\ \text{Pa} + 1.013 \times 10^5\ \text{Pa} = 2.04 \times 10^5\ \text{Pa}.$$

The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = \left(2.04 \times 10^5\ \text{Pa}\right)\left(0.14\,\text{m}^3\right) \ln\left(\frac{2.04 \times 10^5\ \text{Pa}}{1.013 \times 10^5\ \text{Pa}}\right) = 2.00 \times 10^4\ \text{J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$W = p_f\left(V_i - \frac{p_i V_i}{p_f}\right) = \left(p_f - p_i\right)V_i = \left(1.013\times10^5\ \text{Pa} - 2.04\times10^5\ \text{Pa}\right)\left(0.14\,\text{m}^3\right)$$
$$= -1.44\times10^4\ \text{J}.$$

The total work done by the gas over the entire process is

$$W = 2.00 \times 10^4\ \text{J} - 1.44 \times 10^4\ \text{J} = 5.60 \times 10^3\ \text{J}.$$

19. According to kinetic theory, the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where T is the temperature and M is the molar mass. See Eq. 19-34. According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(2.7\,\text{K}\right)}{2.02 \times 10^{-3}\ \text{kg/mol}}} = 1.8\times10^2\ \text{m/s}.$$

21. Table 19-1 gives M = 28.0 g/mol for nitrogen. This value can be used in Eq. 19-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With T_2 = (20.0 + 273.15) K ≈ 293 K, we obtain

$$v_2 = \left(517\,\text{m/s}\right)\sqrt{\frac{293\,\text{K}}{300\,\text{K}}} = 511\,\text{m/s}.$$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = \left(293\,\text{K}\right)\left(\frac{1}{2}\right)^2 = 73.0\,\text{K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\mathrm{K})(4) = 1.17\times 10^3\ \mathrm{K}$$

which is equivalent to 899°C.

35. (a) The average speed is $\bar{v} = \frac{\sum v}{N}$, where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\mathrm{km/s}}{10} = 6.5\,\mathrm{km/s}.$$

(b) The rms speed is given by $v_{\mathrm{rms}} = \sqrt{\frac{\sum v^2}{N}}$. Now

$$\begin{aligned}\sum v^2 &= [(2.0)^2+(3.0)^2+(4.0)^2+(5.0)^2+(6.0)^2\\ &\quad +(7.0)^2+(8.0)^2+(9.0)^2+(10.0)^2+(11.0)^2]\ \mathrm{km^2/s^2} = 505\ \mathrm{km^2/s^2}\end{aligned}$$

so

$$v_{\mathrm{rms}} = \sqrt{\frac{505\ \mathrm{km^2/s^2}}{10}} = 7.1\,\mathrm{km/s}.$$

41. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\,dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 19-24. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0\ ,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) The average speed is given by $v_{\mathrm{avg}} = \int vP(v)\,dv.$ For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{\text{avg}}}{v_0} = 1.22 \,.$$

(c) The mean-square speed is given by $v_{\text{rms}}^2 = \int v^2 P(v)\,dv.$ The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3 dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2 dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \quad \Rightarrow \quad \frac{v_{\text{rms}}}{v_0} = 1.31 \,.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $N\,a(2.0v_0 - 1.5v_0) = 0.5N\,av_0 = N/3$, where $2/3v_0$ was substituted for a. In other words, the fraction of particles in this range is 1/3 or 0.33.

45. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1C_1\,\Delta T$, the internal energy of the second gas changes by $n_2C_2\,\Delta T$, and the internal energy of the third gas changes by $n_3C_3\,\Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1\,C_1 + n_2\,C_2 + n_3\,C_3)\,\Delta T.$$

This must be $(n_1 + n_2 + n_3)\,C_V\,\Delta T$, where C_V is the molar specific heat of the mixture. Thus

$$C_V = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}.$$

With n_1=2.40 mol, C_{V1}=12.0 J/mol·K for gas 1, n_2=1.50 mol, C_{V2}=12.8 J/mol·K for gas 2, and n_3=3.20 mol, C_{V3}=20.0 J/mol·K for gas 3, we obtain C_V =15.8 J/mol·K for the mixture.

46. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the *pV* diagram — the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N / m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int}\,c} - E_{\text{int}\,a} &= \frac{5}{2}\left(p_cV_c - p_aV_a\right) = \frac{5}{2}\left(\left(2.0\times10^3\text{ Pa}\right)\left(4.0\,\text{m}^3\right) - \left(5.0\times10^3\text{ Pa}\right)\left(2.0\,\text{m}^3\right)\right) \\ &= -5.0\times10^3\text{ J.} \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)\left(V_c - V_a\right) = \left(3.5\times10^3\text{ Pa}\right)\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7.0\times10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0\times10^3 + 7.0\times10^3)\text{ J} = 2.0\times10^3\text{ J.}$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{\text{int}} = E_{\text{int}\,c} - E_{\text{int}\,a} = -5.0\times10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = \left(5.0\times10^3\text{ Pa}\right)\left(2.0\,\text{m}^3\right) + 0 = 1.0\times10^4\text{ J.}$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0\times10^3 + 1.0\times10^4)\ \text{J} = 5.0\times10^3\ \text{J}.$$

53. (a) Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R.$ Thus,

$$Q = \frac{7}{2}nR\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times10^3\,\text{J}.$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\,\text{J/mol.K})(60.0\,\text{K}) = 4.99\times10^3\,\text{J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98\times10^3\,\text{J} - 4.99\times10^3\,\text{J} = 1.99\times10^3\,\text{J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times10^3\,\text{J}.$$

67. In this solution we will use non-standard notation: writing ρ for *weight*-density (instead of mass-density), where ρ_c refers to the cool air and ρ_h refers to the hot air. Then the condition required by the problem is

$$F_{\text{net}} = F_{\text{buoyant}} - \text{hot-air-weight} - \text{balloon-weight}$$

$$2.67\times10^3\ \text{N} = \rho_c V - \rho_h V - 2.45\times10^3\ \text{N}$$

where $V = 2.18\times10^3\ \text{m}^3$ and $\rho_c = 11.9\ \text{N/m}^3$. This condition leads to $\rho_h = 9.55\ \text{N/m}^3$. Using the ideal gas law to write ρ_h as PMg/RT where $P = 101000$ Pascals and $M = 0.028$ kg/m^3 (as suggested in the problem), we conclude that the temperature of the enclosed air should be 349 K.

69. (a) By Eq. 19-28, $W = -374$ J (since the process is an adiabatic compression).

(b) $Q = 0$ since the process is adiabatic.

(c) By first law of thermodynamics, the change in internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is $\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}$ J.

71. This is very similar to Sample Problem 19-4 (and we use similar notation here) except for the use of Eq. 19-31 for v_{avg} (whereas in that Sample Problem, its value was just assumed). Thus,

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{p\,d^2}{k}\sqrt{\frac{16\pi R}{MT}} \; .$$

Therefore, with $p = 2.02 \times 10^3$ Pa, $d = 290 \times 10^{-12}$ m and $M = 0.032$ kg/mol (see Table 19-1), we obtain $f = 7.03 \times 10^9\ \text{s}^{-1}$.

77. (a) The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm},$$

(b) For the isothermal process the final temperature of the gas is $T_f = T_i = 300$ K.

(c) The work done is

$$W = nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = (32\,\text{atm})(1.01\times10^5\ \text{Pa/atm})(1.0\times10^{-3}\ \text{m}^3)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right)$$
$$= 4.4\times10^3\ \text{J}.$$

For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

(d) The final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K} \; .$$

(f) The work done is

$$
\begin{aligned}
W &= Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2} nR\Delta T = -\frac{3}{2}\left(p_f V_f - p_i V_i\right) \\
&= -\frac{3}{2}\left[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) \\
&= 2.9\times10^3\ \text{J}\ .
\end{aligned}
$$

If the gas is diatomic, then $\gamma = 1.4$.

(g) The final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^{\gamma} = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\,\text{atm}\ .$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\,\text{K}\ .$$

(i) The work done is

$$
\begin{aligned}
W &= Q - \Delta E_{\text{int}} = -\frac{5}{2} nR\Delta T = -\frac{5}{2}\left(p_f V_f - p_i V_i\right) \\
&= -\frac{5}{2}\left[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) \\
&= 3.4\times10^3\ \text{J}.
\end{aligned}
$$

81. It is recommended to look over §19-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_o} P(v)\,dv = 1 \Rightarrow C = \frac{3}{v_o^3}.$$

(b) The average speed is

$$\int_0^{v_o} vP(v)\,dv = \int_0^{v_o} v\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{4}v_o\ .$$

(c) The rms speed is the square root of

$$\int_0^{v_o} v^2 \; P(v)\, dv = \int_0^{v_o} v^2 \left(\frac{3v^2}{v_o^3}\right) dv = \frac{3}{5} v_o^2 \,.$$

Therefore, $v_{\rm rms} = \sqrt{3/5}\, v_o \approx 0.775 v_o$.

83. (a) The temperature is 10.0°C → T = 283 K. Then, with n = 3.50 mol and V_f/V_0 = 3/4, we use Eq. 19-14:

$$W = nRT \, \ln\left(\frac{V_f}{V_0}\right) = -2.37\,\text{kJ}.$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.37$ kJ. The negative value implies that the heat transfer is from the sample to its environment.

5. (a) Since the gas is ideal, its pressure p is given in terms of the number of moles n, the volume V, and the temperature T by $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p\,dV = n\,RT \int_{V_1}^{V_2} \frac{dV}{V} = n\,RT \ln\frac{V_2}{V_1}\ .$$

We substitute $V_2 = 2.00V_1$ to obtain

$$W = n\,RT \ln 2.00 = (4.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(400 \text{ K})\ln 2.00 = 9.22\times10^3 \text{ J}.$$

(b) Since the expansion is isothermal, the change in entropy is given by

$$\Delta S = \int (1/T)\ dQ = Q/T\ ,$$

where Q is the heat absorbed. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J/K}.$$

(c) $\Delta S = 0$ for all reversible adiabatic processes.

7. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a \left(T_{ai} - T_f\right) = m_w c_w \left(T_f - T_{wi}\right) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$T_f = \frac{(0.200 \text{ kg})(900 \text{ J/kg}\cdot\text{K})(100°\text{C}) + (0.0500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(20°\text{C})}{(0.200 \text{ kg})(900 \text{ J/kg}\cdot\text{K}) + (0.0500 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})} = 57.0°\text{C} = 330 \text{ K}.$$

(b) Now temperatures must be given in Kelvins: T_{ai} = 393 K, T_{wi} = 293 K, and T_f = 330 K. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\Delta S_a = \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} = (0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})\ln\left(\frac{330\text{ K}}{373\text{ K}}\right) = -22.1\text{ J/K}.$$

(c) The entropy change for the water is

$$\Delta S_w = \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} = (0.0500\text{ kg})(4190\text{ J/kg.K})\ln\left(\frac{330\,\text{K}}{293\,\text{K}}\right) = +24.9\,\text{J/K}.$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1\text{ J/K} + 24.9\text{ J/K} = +2.8\text{ J/K}.$$

25. (a) The efficiency is

$$\varepsilon = \frac{T_\text{H} - T_\text{L}}{T_\text{H}} = \frac{(235-115)\,\text{K}}{(235+273)\,\text{K}} = 0.236 = 23.6\% .$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_\text{H}|$, the work done is given by

$$|W| = \varepsilon |Q_\text{H}| = 0.236(6.30\times10^4\text{ J}) = 1.49\times10^4\text{ J} .$$

29. (a) Energy is added as heat during the portion of the process from a to b. This portion occurs at constant volume (V_b), so $Q_\text{in} = nC_V\,\Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a) V_b.$$

Thus, $Q_\text{in} = \frac{3}{2}(p_b - p_a)V_b$. V_b and p_b are given. We need to find p_a. Now p_a is the same as p_c and points c and b are connected by an adiabatic process. Thus, $p_c V_c^\gamma = p_b V_b^\gamma$ and

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3}(1.013\times10^6\text{ Pa}) = 3.167\times10^4\text{ Pa}.$$

The energy added as heat is

$$Q_{\text{in}} = \frac{3}{2}\left(1.013\times10^{6}\ \text{Pa} - 3.167\times10^{4}\ \text{Pa}\right)\left(1.00\times10^{-3}\ \text{m}^3\right) = 1.47\times10^{3}\ \text{J}.$$

(b) Energy leaves the gas as heat during the portion of the process from *c* to *a*. This is a constant pressure process, so

$$Q_{\text{out}} = nC_p\Delta T = \frac{5}{2}\left(p_aV_a - p_cV_c\right) = \frac{5}{2}p_a\left(V_a - V_c\right)$$
$$= \frac{5}{2}\left(3.167\times10^{4}\ \text{Pa}\right)\left(-7.00\right)\left(1.00\times10^{-3}\ \text{m}^3\right) = -5.54\times10^{2}\ \text{J},$$

or $|Q_{\text{out}}| = 5.54\times10^{2}\ \text{J}$. The substitutions $V_a - V_c = V_a - 8.00\ V_a = -7.00\ V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = 1.47 \times 10^3\ \text{J} - 5.54 \times 10^2\ \text{J} = 9.18 \times 10^2\ \text{J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_{\text{in}} = (9.18 \times 10^2\ \text{J})/(1.47 \times 10^3\ \text{J}) = 0.624 = 62.4\%.$$

37. A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance *K* that is given by

$$K = \frac{T_L}{T_H - T_L}.$$

For the refrigerator of this problem, T_H = 96° F = 309 K and T_L = 70° F = 294 K, so *K* = (294 K)/(309 K – 294 K) = 19.6. The coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$. Thus,

$$|Q_L| = K|W| = (19.6)(1.0\ \text{J}) = 20\ \text{J}.$$

39. The coefficient of performance for a refrigerator is given by $K = |Q_L|/|W|$, where Q_L is the energy absorbed from the cold reservoir as heat and *W* is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than *W*, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$. In one hour,

$$|W| = \frac{7.54\,\text{MJ}}{3.8+1} = 1.57\,\text{MJ}.$$

The rate at which work is done is (1.57×10^6 J)/(3600 s) = 440 W.

47. (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are N! arrangements of the N molecules, but n_L! are simply rearrangements of the n_L molecules in the right third, n_C! are rearrangements of the n_C molecules in the center third, and n_R! are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L!\,n_C!\,n_R!}.$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!\,(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!\,(N/3)!\,(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!\,(N/2)!}{(N/3)!\,(N/3)!\,(N/3)!}.$$

(c) For N = 100,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.2\times10^{16}.$$

49. Using Eq. 19-34 and Eq. 19-35, we arrive at

$$\Delta v = (\sqrt{3} - \sqrt{2})\sqrt{RT/M}$$

(a) We find, with M = 28 g/mol = 0.028 kg/mol (see Table 19-1), Δv_i=87 m/s at 250 K,

(b) and Δv_f=122 $\approx 1.2\times10^2$ m/s at 500 K.

(c) The expression above for Δv implies

$$T = \frac{M}{R(\sqrt{3} - \sqrt{2})^2} (\Delta v)^2$$

which we can plug into Eq. 20-4 to yield

$$\Delta S = nR \ln(V_f / V_i) + nC_V \ln(T_f / T_i) = 0 + nC_V \ln[(\Delta v_f)^2 / (\Delta v_i)^2] = 2nC_V \ln(\Delta v_f / \Delta v_i).$$

Using Table 19-3 to get $C_V = 5R/2$ (see also Table 19-2) we then find, for $n = 1.5$ mol, $\Delta S = 22$ J/K.

55. Except for the phase change (which just uses Eq. 20-2), this has some similarities with Sample Problem 20-2. Using constants available in the Chapter 19 tables, we compute

$$\Delta S = m[c_{\text{ice}} \ln(273/253) + \frac{L_f}{273} + c_{\text{water}} \ln(313/273)] = 1.18 \times 10^3 \text{ J/K}.$$

63. (a) Eq. 20-15 can be written as $|Q_H| = |Q_L|(1 + 1/K_C) = (35)(1 + \frac{1}{4.6}) = 42.6$ kJ.

(b) Similarly, Eq. 20-14 leads to $|W| = |Q_L|/K = 35/4.6 = 7.61$ kJ.

67. We adapt the discussion of §20-7 to 3 and 5 particles (as opposed to the 6 particle situation treated in that section).

(a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, using Eq. 20-20, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box B, $W = 5!/(5!0!) = 1$ in the "least" case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Thus,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k \ln W = \left(1.38\times10^{-23}\right)\ln 3 = 1.5\times10^{-23} \text{ J/K.}$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k \ln 10 = 3.2 \times 10^{-23} \text{ J/K.}$$

Chapter 21

1

The magnitude of the force that either charge exerts on the other is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2},$$

where r is the distance between them. Thus

$$\begin{aligned} r &= \sqrt{\frac{|q_1||q_2|}{4\pi\epsilon_0 F}} \\ &= \sqrt{\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(26.0\times10^{-6}\,\mathrm{C})(47.0\times10^{-6}\,\mathrm{C})}{5.70\,\mathrm{N}}} = 1.38\,\mathrm{m}\,. \end{aligned}$$

5

The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{q(Q-q)}{r^2},$$

where r is the distance between the charges. You want the value of q that maximizes the function $f(q) = q(Q-q)$. Set the derivative df/dq equal to zero. This yields $Q - 2q = 0$, or $q = Q/2$.

7

Assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges and choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Take the distance between the charges to be r. Then the force on q_2 is

$$F_a = -\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}.$$

The negative sign indicates that the spheres attract each other.

After the wire is connected, the spheres, being identical, have the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1+q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\epsilon_0}\frac{(q_1+q_2)^2}{4r^2}.$$

Solve the two force equations simultaneously for q_1 and q_2. The first gives

$$q_1 q_2 = -4\pi\epsilon_0 r^2 F_a = -\frac{(0.500\,\mathrm{m})^2(0.108\,\mathrm{N})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = -3.00\times10^{-12}\,\mathrm{C^2}$$

and the second gives

$$q_1 + q_2 = 2r\sqrt{4\pi\epsilon_0 F_b} = 2(0.500\,\mathrm{m})\sqrt{\frac{0.0360\,\mathrm{N}}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}}} = 2.00\times10^{-6}\,\mathrm{C}\,.$$

Thus

$$q_2 = \frac{-(3.00\times10^{-12}\,\mathrm{C^2})}{q_1}$$

and substitution into the second equation gives

$$q_1 + \frac{-3.00\times10^{-12}\,\mathrm{C^2}}{q_1} = 2.00\times10^{-6}\,\mathrm{C}\,.$$

Multiply by q_1 to obtain the quadratic equation

$$q_1^2 - (2.00\times10^{-6}\,\mathrm{C})q_1 - 3.00\times10^{-12}\,\mathrm{C^2} = 0\,.$$

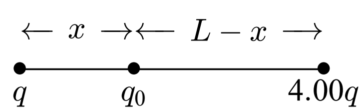
The solutions are

$$q_1 = \frac{2.00\times10^{-6}\,\mathrm{C} \pm \sqrt{(-2.00\times10^{-6}\,\mathrm{C})^2 + 4(3.00\times10^{-12}\,\mathrm{C^2})}}{2}\,.$$

If the positive sign is used, $q_1 = 3.00\times10^{-6}\,\mathrm{C}$ and if the negative sign is used, $q_1 = -1.00\times10^{-6}\,\mathrm{C}$. Use $q_2 = (-3.00\times10^{-12})/q_1$ to calculate q_2. If $q_1 = 3.00\times10^{-6}\,\mathrm{C}$, then $q_2 = -1.00\times10^{-6}\,\mathrm{C}$ and if $q_1 = -1.00\times10^{-6}\,\mathrm{C}$, then $q_2 = 3.00\times10^{-6}\,\mathrm{C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00\times10^{-6}\,\mathrm{C}$ and the other had charge $+3.00\times10^{-6}\,\mathrm{C}$.

19

If the system of three particles is to be in equilibrium, the force on each particle must be zero. Let the charge on the third particle be q_0. The third particle must lie on the x axis since otherwise the two forces on it would not be along the same line and could not sum to zero. Thus the y coordinate of the particle must be zero. The third particle must lie between the other two since otherwise the forces acting on it would be in the same direction and would not sum to zero. Suppose the third particle is a distance x from the particle with charge q, as shown on the diagram to the right. The force acting on it is then given by



$$F_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} - \frac{4.00qq_0}{(L-x)^2}\right] = 0\,,$$

where the positive direction was taken to be toward the right. Solve this equation for x. Canceling common factors yields $1/x^2 = 4.00/(L-x)^2$ and taking the square root yields $1/x = 2.00/(L-x)$. The solution is $x = 0.333L$.

The force on q is

$$F_q = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} + \frac{4.00q^2}{L^2}\right] = 0\,.$$

Solve for q_0: $q_0 = -4.00qx^2/L^2 = -0.444q$, where $x = 0.333L$ was used.

The force on the particle with charge $4.00q$ is

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00qq_0}{(L-x)^2}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00(0.444)q^2}{(0.444)L^2}\right] \\ &= \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} - \frac{4.00q^2}{L^2}\right] = 0\,. \end{aligned}$$

With $q_0 = -0.444q$ and $x = 0.333L$, all three charges are in equilibrium.

<u>25</u>

(a) The magnitude of the force between the ions is given by

$$F = \frac{q^2}{4\pi\epsilon_0 r^2}\,,$$

where q is the charge on either of them and r is the distance between them. Solve for the charge:

$$q = r\sqrt{4\pi\epsilon_0 F} = (5.0\times 10^{-10}\,\mathrm{m})\sqrt{\frac{3.7\times 10^{-9}\,\mathrm{N}}{8.99\times 10^{9}\,\mathrm{N\cdot m^2/C^2}}} = 3.2\times 10^{-19}\,\mathrm{C}\,.$$

(b) Let N be the number of electrons missing from each ion. Then $Ne = q$ and

$$N = \frac{q}{e} = \frac{3.2\times 10^{-19}\,\mathrm{C}}{1.60\times 10^{-19}\,\mathrm{C}} = 2\,.$$

<u>35</u>

(a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion and, as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus the distance from the center of the cube to a corner is $d = (\sqrt{3}/2)a$. The force has magnitude

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(3/4)a^2} \\ &= \frac{(8.99\times 10^{9}\,\mathrm{N\cdot m^2/C^2})(1.60\times 10^{-19}\,\mathrm{C})^2}{(3/4)(0.40\times 10^{-9}\,\mathrm{m})^2} = 1.9\times 10^{-9}\,\mathrm{N}\,. \end{aligned}$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pulled away from the site of the missing cesium ion.

37

None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

(a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and $9 - 4 = 5$ neutrons, so X has $1 + 4 = 5$ protons, $1 + 4 = 5$ electrons, and $0 + 5 - 1 = 4$ neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of $5\,\text{g/mol} + 4\,\text{g/mol} = 9\,\text{g/mol}$: ^{9}B.

(b) ^{12}C has 6 protons, 6 electrons, and $12 - 6 = 6$ neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has $6 + 1 = 7$ protons, $6 + 1 = 7$ electrons, and $6 + 0 = 6$ neutrons. It must be nitrogen with a molar mass of $7\,\text{g/mol} + 6\,\text{g/mol} = 13\,\text{g/mol}$: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and $15 - 7 = 8$ neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and $4 - 2 = 2$ neutrons; so X has $7 + 1 - 2 = 6$ protons, 6 electrons, and $8 + 0 - 2 = 6$ neutrons. It must be carbon with a molar mass of $6\,\text{g/mol} + 6\,\text{g/mol} = 12\,\text{g/mol}$: ^{12}C.

39

The magnitude of the force of particle 1 on particle 2 is

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{d_1^2 + d_2^2}\,.$$

The signs of the charges are the same, so the particles repel each other along the line that runs through them. This line makes an angle θ with the x axis such that $\cos\theta = d_2/\sqrt{d_1^2 + d_2^2}$, so the x component of the force is

$$\begin{aligned} F_x &= \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{d_1^2 + d_2^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|d_2}{(d_1^2 + d_2^2)^{3/2}} \\ &= (8.99 \times 10^9\,\text{C}^2/\text{N}\cdot\text{m}^2)\frac{24(1.60 \times 10^{-19}\,\text{C})^2(6.00 \times 10^{-3}\,\text{m})}{[(2.00 \times 10^{-3}\,\text{m})^2 + (6.00 \times 10^{-3}\,\text{m})^2]^{3/2}} = 1.31 \times 10^{-22}\,\text{N}\,. \end{aligned}$$

50

The magnitude of the gravitational force on a proton near Earth's surface is mg, where m is the mass of the proton (1.67×10^{-27} kg from Appendix B). The electrostatic force between two protons is $F = (1/4\pi\epsilon_0)(e^2/d^2)$, where d is their separation. Equate these forces to each other and solve for d. The result is

$$d = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mg}} = \sqrt{(8.99 \times 10^{-9}\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\,\text{C})^2}{(1.67 \times 10{-}27\,\text{kg})(9.8\,\text{m/s}^2)}} = 0.119\,\text{m}\,.$$

60

The magnitude of the force of particle 1 on particle 4 is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_4|}{d_1^2} = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-19}\,\text{C})(3.20\times10^{-19}\,\text{C})}{(0.0300\,\text{m})^2} = 1.02\times10^{-24}\,\text{N}\,.$$

The charges have opposite signs, so the particles attract each other and the vector force is

$$\begin{aligned}\vec{F}_1 &= -(1.02^{-24}\,\text{N})(\cos 35.0^\circ)\,\hat{\text{i}} - (1.02\times10^{-24}\,\text{N})(\sin 35.0^\circ)\,\hat{\text{j}}\\ &= -(8.36\times10^{-25}\,\text{N})\,\hat{\text{i}} - (5.85\times10^{-25}\,\text{N})\,\hat{\text{j}}\,.\end{aligned}$$

Particles 2 and 3 repel each other. The force of particle 2 on particle 4 is

$$\begin{aligned}\vec{F}_2 &= -\frac{1}{4\pi\epsilon_0}\frac{|q_2||q_4|}{d_2^2}\,\hat{\text{j}}\\ &= -(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-19}\,\text{C})(3.20\times10^{-19}\,\text{C})}{(0.0200\,\text{m})^2}\,\hat{\text{j}} = -(2.30\times10^{-24}\,\text{N})\,\hat{\text{j}}\,.\end{aligned}$$

Particles 3 and 4 repel each other and the force of particle 3 on particle 4 is

$$\begin{aligned}\vec{F}_3 &= -\frac{1}{4\pi\epsilon_0}\frac{|2q_3||q_4|}{d_3^2}\,\hat{\text{i}}\\ &= -(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.40\times10^{-19}\,\text{C})(3.20\times10^{-19}\,\text{C})}{(0.0200\,\text{m})^2} = -(4.60\times10^{-24}\,\text{C})\,\hat{\text{i}}\,.\end{aligned}$$

The net force is the vector sum of the three forces. The x component is $F_x = -18.36\times10^{-25}\,\text{N} - 4.60\,\text{N} = -5.44\times10^{-24}\,\text{N}$ and the y component is $F_y = -5.85\times10^{-25}\,\text{N} - 2.30\times10^{-24}\,\text{N} = -2.89\times10^{-24}\,\text{N}$. The magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-5.44\times10^{-24}\,\text{N})^2 + (-2.89\times10^{-24}\,\text{N})^2} = 6.16\times10^{24}\,\text{N}\,.$$

The tangent of the angle θ between the net force and the positive x axis is $tan\theta = F_y/F_x = (-2.89\times10^{-24}\,\text{N})/(-5.44\times10^{-24}\,\text{N}) = 0.531$ and the angle is either 28° or 208°. The later angle is associated with a vector that has negative x and y components and so is the correct angle.

69

The net force on particle 3 is the vector sum of the forces of particles 1 and 2 and for this to be zero the two forces must be along the same line. Since electrostatic forces are along the lines that join the particles, particle 3 must be on the x axis. Its y coordinate is zero.

Particle 3 is repelled by one of the other charges and attracted by the other. As a result, particle 3 cannot be between the other two particles and must be either to the left of particle 1 or to the right of particle 2. Since the magnitude of q_1 is greater than the magnitude of q_2, particle 3 must be closer to particle 2 than to particle 1 and so must be to the right of particle 2. Let x be the coordinate of particle 3. The the x component of the force on it is

$$F_x = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{x^2} + \frac{q_2 q_3}{(x-L)^2}\right].$$

If $F_x = 0$ the solution for x is

$$x = \frac{\sqrt{-q_1/q_2}}{\sqrt{-q_1/q_2}-1}L = \frac{\sqrt{-(-5.00q)/(2.00q)}}{\sqrt{-(-5.00q)/(2.00q)}-1}L = 2.72L\,.$$

Chapter 22

3

Since the magnitude of the electric field produced by a point particle with charge q is given by $E = |q|/4\pi\epsilon_0 r^2$, where r is the distance from the particle to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\epsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times10^{-11}\,\text{C}\,.$$

5

Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\epsilon_0 R^2}\,,$$

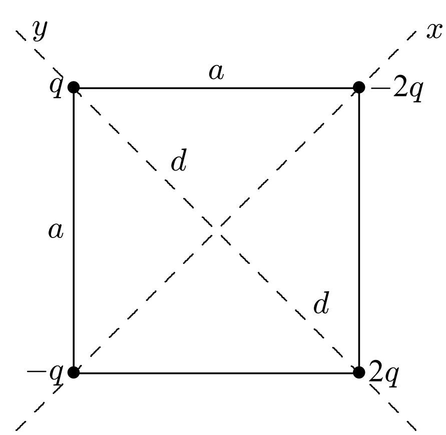
where q is the magnitude of the total charge and R is the sphere radius. The magnitude of the total charge is Ze, so

$$E = \frac{Ze}{4\pi\epsilon_0 R^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(94)(1.60\times10^{-19}\,\text{C})}{(6.64\times10^{-15}\,\text{m})^2} = 3.07\times10^{21}\,\text{N/C}\,.$$

The field is normal to the surface and since the charge is positive it points outward from the surface.

9

Choose the coordinate axes as shown on the diagram to the right. At the center of the square, the electric fields produced by the particles at the lower left and upper right corners are both along the x axis and each points away from the center and toward the particle that produces it. Since each particle is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center, the net field due to these two particles is



$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{a^2/2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-8}\,\text{C})}{(0.050\,\text{m})^2/2} = 7.19\times10^4\,\text{N/C}.$$

At the center of the square, the field produced by the particles at the upper left and lower right corners are both along the y axis and each points away from the particle that produces it. The net field produced at the center by these particles is

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\epsilon_0}\,\frac{q}{a^2/2} = 7.19 \times 10^4\,\mathrm{N/C}\,.$$

The magnitude of the net field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19 \times 10^4\,\mathrm{N/C})^2} = 1.02 \times 10^5\,\mathrm{N/C}$$

and the angle it makes with the x axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45^\circ\,.$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

21

Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is $qd/2\pi\epsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\epsilon_0(z + d/2)^3$. Use the binomial expansions $(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$ and $(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$ to obtain

$$E = \frac{qd}{2\pi\epsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\epsilon_0 z^4}\,.$$

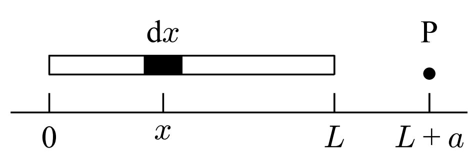
Let $Q = 2qd^2$. Then

$$E = \frac{3Q}{4\pi\epsilon_0 z^4}\,.$$

27

(a) The linear charge density λ is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L = -(4.23 \times 10^{-15}\,\mathrm{C})/(0.0815\,\mathrm{m}) = -5.19 \times 10^{-14}\,\mathrm{C/m}$.

(b) and (c) Position the origin at the left end of the rod, as shown in the diagram. Let $\mathrm{d}x$ be an infinitesimal length of rod at x. The charge in this segment is $\mathrm{d}q = \lambda\,\mathrm{d}x$. Since the segment may be taken to be a point particle, the electric field it produces at point P has only an x component and this component is given by



$$\mathrm{d}E_x = \frac{1}{4\pi\epsilon_0}\,\frac{\lambda\,\mathrm{d}x}{(L + a - x)^2}\,.$$

The total electric field produced at P by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{\mathrm{d}x}{(L+a-x)^2} = \frac{\lambda}{4\pi\epsilon_0}\left.\frac{1}{L+a-x}\right|_0^L$$
$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{L+a}\right] = \frac{\lambda}{4\pi\epsilon_0}\frac{L}{a(L+a)}\,.$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\epsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4.23\times10^{-15}\,\mathrm{C})}{(0.120\,\mathrm{m})(0.0815\,\mathrm{m}+0.120\,\mathrm{m})} = -1.57\times10^{-3}\,\mathrm{N/C}\,.$$

The negative sign indicates that the field is toward the rod and makes an angle of 180° with the positive x direction.

(d) Now

$$E_x = -\frac{1}{4\pi\epsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4.23\times10^{-15}\,\mathrm{C})}{(50\,\mathrm{m})(0.0815\,\mathrm{m}+50\,\mathrm{m})} = -1.52\times10^{-8}\,\mathrm{N/C}\,.$$

(e) The field of a point particle at the origin is

$$E_x = -\frac{q}{4\pi\epsilon_0 a^2} = -\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4.23\times10^{-15}\,\mathrm{C})}{(50\,\mathrm{m})^2} = -1.52\times10^{-8}\,\mathrm{N/C}\,.$$

35

At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right],$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22–26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\epsilon_0$. You want to solve for the value of z such that $E/E_c = 1/2$. This means

$$\frac{E}{E_c} = 1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}$$

or

$$\frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}\,.$$

Square both sides, then multiply them by z^2+R^2 to obtain $z^2 = (z^2/4)+(R^2/4)$. Thus $z^2 = R^2/3$ and $z = R/\sqrt{3} = (0.600\,\mathrm{m})/\sqrt{3} = 0.346\,\mathrm{m}$.

39

The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60\times10^{-19}\,\mathrm{C})(2.00\times10^4\,\mathrm{N/C})}{9.11\times10^{-31}\,\mathrm{kg}} = 3.51\times10^{15}\,\mathrm{m/s^2}\,.$$

__43__

(a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6}\,\mathrm{N}}{2.0 \times 10^{-9}\,\mathrm{C}} = 1.5 \times 10^{3}\,\mathrm{N/C}\,.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60 \times 10^{-19}\,\mathrm{C})(1.5 \times 10^{3}\,\mathrm{N/C}) = 2.4 \times 10^{-16}\,\mathrm{N}\,.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67 \times 10^{-27}\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 1.64 \times 10^{-26}\,\mathrm{N}\,.$$

The force is downward.

(e) The ratio of the force magnitudes is

$$\frac{F_e}{F_g} = \frac{2.4 \times 10^{-16}\,\mathrm{N}}{1.64 \times 10^{-26}\,\mathrm{N}} = 1.5 \times 10^{10}\,.$$

__45__

(a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus

$$a = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(2.00 \times 10^{4}\,\mathrm{N/C})}{1.67 \times 10^{-27}\,\mathrm{kg}} = 1.92 \times 10^{12}\,\mathrm{m/s^2}\,.$$

(b) Assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2(1.92 \times 10^{12}\,\mathrm{m/s^2})(0.0100\,\mathrm{m})} = 1.96 \times 10^{5}\,\mathrm{m/s}\,.$$

__57__

(a) If q is the positive charge in the dipole and d is the separation of the charged particles, the magnitude of the dipole moment is $p = qd = (1.50 \times 10^{-9}\,\mathrm{C})(6.20 \times 10^{-6}\,\mathrm{m}) = 9.30 \times 10^{-15}\,\mathrm{C \cdot m}$.

(b) If the initial angle between the dipole moment and the electric field is θ_0 and the final angle is θ, then the change in the potential energy as the dipole swings from $\theta = 0$ to $\theta = 180°$ is

$$\begin{aligned}\Delta U &= -pE(\cos\theta - \cos\theta_0) = -(9.30 \times 10^{-15}\,\mathrm{C \cdot m})(1100\,\mathrm{N/C})(\cos 180° - \cos 0)\\ &= 2.05 \times 10^{-11}\,\mathrm{J}\,.\end{aligned}$$

79

(a) and (b) Since the field at the point on the x axis with coordinate $x = 2.0\,\text{cm}$ is in the positive x direction you know that the charged particle is on the x axis. The line through the point with coordinates $x = 3.0\,\text{cm}$ and $y = 3.0\,\text{cm}$ and parallel to the field at that point must pass through the position of the particle. Such a line has slope $(3.0)/(4.0) = 0.75$ and its equation is $y = 0.57 + (0.75)x$. The solution for $y = 0$ is $x = -1.0\,\text{cm}$, so the particle is located at the point with coordinates $x = -1.0\,\text{cm}$ and $y = 0$.

(c) The magnitude of the field at the point on the x axis with coordinate $x = 2.0\,\text{cm}$ is given by $E = (1/4\pi\epsilon_0)q/(2.0\,\text{cm} - x)^2$, so

$$q = 4\pi\epsilon_0 x^2 E = \frac{(0.020\,\text{m} + 0.010\,\text{m})^2(100\,\text{N/C})}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 1.0 \times 10^{-11}\,\text{C}\,.$$

81

(a) The potential energy of an electric dipole with dipole moment $\vec{p}$ in an electric field $\vec{E}$ is

$$\begin{aligned} U = -\vec{p}\cdot\vec{E} &= (1.24 \times 10^{-30}\,\text{C}\cdot\text{m})(3.00\,\hat{\text{i}} + 4.00\,\hat{\text{j}})\cdot(4000\,\text{N/C})\,\hat{\text{i}} \\ &= -(1.24 \times 10^{-30}\,\text{C}\cdot\text{m})(3.00)(4000\,\text{N/C}) = -1.49 \times 10^{-26}\,\text{J}\,. \end{aligned}$$

Here we used $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$ to evaluate the scalar product.

(b) The torque is

$$\begin{aligned} \vec{\tau} = \vec{p}\times\vec{E} &= (p_x\,\hat{\text{i}} + p_y\,\hat{\text{j}}) \times (E_x\,\hat{\text{i}}) = -p_y E_x\,\hat{\text{k}} \\ &= -(4.00)(1.24 \times 10^{-30}\,\text{C}\cdot\text{m})(4000\,\text{N/C}) = -(1.98 \times 10^{-26}\,\text{N}\cdot\text{m})\,\hat{\text{k}}\,. \end{aligned}$$

(c) The work done by the agent is equal to the change in the potential energy of the dipole. The initial potential energy is $U_i = -1.49 \times 10^{-26}\,\text{J}$, as computed in part (a). The final potential energy is

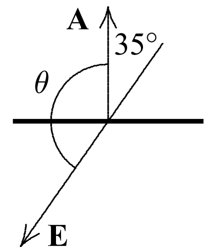
$$\begin{aligned} U_f &= (1.24 \times 10^{-30}\,\text{C}\cdot\text{m})(-4.00\,\hat{\text{i}} + 3.00\,\hat{\text{j}})\cdot(4000\,\text{N/C})\,\hat{\text{i}} \\ &= -(1.24 \times 10^{-30}\,\text{C}\cdot\text{m})(-4.00)(4000\,\text{N/C}) = +1.98 \times 10^{-26}\,\text{J}\,. \end{aligned}$$

The work done by the agent is $W = (1.98 \times 10^{-26}\,\text{J}) - (-1.49 \times 10^{-26}\,\text{J}) = 3.47 \times 10^{-26}\,\text{J}$.

Chapter 23

<u>1</u>

The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram to the right. The angle θ between them is $180° - 35° = 145°$, so the electric flux through the area is $\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta = (1800\,\mathrm{N/C})(3.2 \times 10^{-3}\,\mathrm{m})^2 \cos 145° = -1.5 \times 10^{-2}\,\mathrm{N \cdot m^2/C}$.



<u>9</u>

Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$q = \epsilon_0\Phi = \epsilon_0 A(E_\ell - E_u) = (8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(100\,\mathrm{m})^2(100\,\mathrm{N/C} - 60.0\,\mathrm{N/C})$$
$$= 3.54 \times 10^{-6}\,\mathrm{C} = 3.54\,\mu\mathrm{C}\,.$$

<u>19</u>

(a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere ($4\pi r^2$, where r is the radius). Thus

$$q = 4\pi r^2\sigma = 4\pi\left(\frac{1.2\,\mathrm{m}}{2}\right)^2 (8.1 \times 10^{-6}\,\mathrm{C/m^2}) = 3.7 \times 10^{-5}\,\mathrm{C}\,.$$

(b) Choose a Gaussian surface in the form a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux through the surface is given by Gauss' law:

$$\Phi = \frac{q}{\epsilon_0} = \frac{3.7 \times 10^{-5}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}} = 4.1 \times 10^6\,\mathrm{N \cdot m^2/C}\,.$$

<u>23</u>

The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\epsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 23–12. Thus

$$\lambda = 2\pi\epsilon_0 Er = 2\pi(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(4.5 \times 10^4\,\mathrm{N/C})(2.0\,\mathrm{m}) = 5.0 \times 10^{-6}\,\mathrm{C/m}\,.$$

27

Assume the charge density of both the conducting rod and the shell are uniform. Neglect fringing. Symmetry can be used to show that the electric field is radial, both between the rod and the shell and outside the shell. It is zero, of course, inside the rod and inside the shell since they are conductors.

(a) and (b) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface outside the shell. The area of the curved surface is $2\pi rL$. The field is normal to the curved portion of the surface and has uniform magnitude over it, so the flux through this portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. The flux through the ends is zero. The charge enclosed by the Gaussian surface is $Q_1 - 2.00Q_1 = -Q_1$. Gauss' law yields $2\pi r\epsilon_0 LE = -Q_1$, so

$$E = -\frac{Q_1}{2\pi\epsilon_0 Lr} = -\frac{(8.99 \times 10^9\, N \cdot \mathrm{m}^2/\mathrm{C}^2)(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(26.0 \times 10^{-3}\,\mathrm{m})} = -0.214\,\mathrm{N/C}\,.$$

The magnitude of the field is $0.214\,\mathrm{N/C}$. The negative sign indicates that the field points inward.

(c) and (d) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface between the conducting rod and the shell. As in (a), the flux through the curved portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface, and the flux through the ends is zero. The charge enclosed by the Gaussian surface is only the charge Q_1 on the conducting rod. Gauss' law yields $2\pi\epsilon_0 rLE = Q_1$, so

$$E = \frac{Q_1}{2\pi\epsilon_0 Lr} = \frac{2(8.99 \times 10^9\,\mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(6.50 \times 10^{-3}\,\mathrm{m})} = +0.855\,\mathrm{N/C}\,.$$

The positive sign indicates that the field points outward.

(e) Consider a Gaussian surface in the form of a cylinder of length L with the curved portion of its surface completely within the shell. The electric field is zero at all points on the curved surface and is parallel to the ends, so the total electric flux through the Gaussian surface is zero and the net charge within it is zero. Since the conducting rod, which is inside the Gaussian cylinder, has charge Q_1, the inner surface of the shell must have charge $-Q_1 = -3.40 \times 10^{-12}\,\mathrm{C}$.

(f) Since the shell has total charge $-2.00Q_1$ and has charge $-Q_1$ on its inner surface, it must have charge $-Q_1 = -3.40 \times 10^{-12}\,\mathrm{C}$ on its outer surface.

35

(a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\epsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\,\mathrm{C/m}^2\,.$$

The magnitude of the field is

$$E = \frac{\sigma}{\epsilon_0} = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}} = 5.3 \times 10^{7}\,\mathrm{N/C}\,.$$

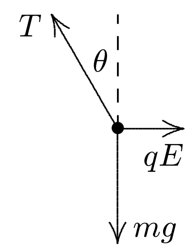
The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\epsilon_0 r^2$, where r is the distance from the plate. Thus

$$E = \frac{(8.99 \times 10^{9}\,\mathrm{N\cdot m^2/C^2})(6.0 \times 10^{-6}\,\mathrm{C})}{(30\,\mathrm{m})^2} = 60\,\mathrm{N/C}\,.$$

41

The forces on the ball are shown in the diagram to the right. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the electric field at the position of the ball; and the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$.

The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\epsilon_0$, where σ is the surface charge density. Thus

$$\frac{q\sigma}{2\epsilon_0} = mg\tan\theta$$

and

$$\begin{aligned}\sigma &= \frac{2\epsilon_0 mg\tan\theta}{q} \\ &= \frac{2(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.0 \times 10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\tan 30^\circ}{2.0 \times 10^{-8}\,\mathrm{C}} \\ &= 5.0 \times 10^{-9}\,\mathrm{C/m^2}\,.\end{aligned}$$

45

Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\epsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus

$$q = 4\pi\epsilon_0 r^2 E = \frac{(0.15\,\text{m})^2(3.0\times10^3\,\text{N/C})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 7.5\times10^{-9}\,\text{C}\,.$$

The field points inward, toward the sphere center, so the charge is negative: -7.5×10^{-9} C.

<u>49</u>

To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance.

Use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law is used to find the magnitude of the electric field a distance r_g from the shell center.

The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_{\text{enc}} = \int \rho\,\mathrm{d}V$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take $\mathrm{d}V$ to be the volume of a spherical shell with radius r and infinitesimal thickness $\mathrm{d}r$: $\mathrm{d}V = 4\pi r^2\,\mathrm{d}r$. Thus

$$q_{\text{enc}} = 4\pi\int_a^{r_g}\rho r^2\,\mathrm{d}r = 4\pi\int_a^{r_g}\frac{A}{r}r^2\,\mathrm{d}r = 4\pi A\int_a^{r_g} r\,\mathrm{d}r = 2\pi A(r_g^2 - a^2)\,.$$

The total charge inside the Gaussian surface is $q + q_{\text{enc}} = q + 2\pi A(r_g^2 - a^2)$.

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\epsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2)\,.$$

Solve for E:

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right]\,.$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2 = (45.0\times10^{-15}\,\text{C})/2\pi(2.00\times10^{-2}\,\text{m})^2 = 1.79\times10^{-11}\,\text{C/m}^2$.

<u>59</u>

(a) The magnitude E_1 of the electric field produced by the charge q on the spherical shell is $E_1 = q/4\pi\epsilon_0 R_o^2$, where R_o is the radius of the outer surface of the shell. Thus

$$q = 4\pi\epsilon_0 E_1 R_o^2 = \frac{(450\,\text{N/C})(0.20\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 2.0\times10^{-9}\,\text{C}\,.$$

(b) Since the field at P is outward and is reduced in magnitude the field of Q must be inward. Q is a negative charge and the magnitude of its field at P is $E_2 = 450\,\text{N/C} - 180\,\text{N/C} = 270\,\text{N/C}$. The value of Q is

$$Q = 4\pi\epsilon_0 E_2 R_o^2 = -\frac{(270\,\text{N/C})(0.20\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C})^2} = -1.2\times10^{-9}\,\text{C}\,.$$

(c) Gauss' law tells us that since the electric field is zero inside a conductor the net charge inside a spherical surface with a radius that is slightly larger than the inside radius of the shell must be zero. Thus the charge on the inside surface of the shell is $+1.2 \times 10^{-9}$ C.

(d) The remaining charge on the shell must be on its outer surface and this is $2.0 \times 10^{-9}\,\text{C} - 1.2 \times 10^{-9}\,\text{C} = +0.8 \times 10^{-9}\,\text{C}$.

69

(a) Draw a spherical Gaussian surface with radius r, concentric with the shells. The electric field, if it exists, is radial and so is normal to the surface. The integral in Gauss' law is $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$, where E is the radial component of the field. For $r < a$ then charge enclosed is zero. Gauss' law gives $4\pi r^2 E = 0$, so $E = 0$.

(b) For $a < r < b$ the charge enclosed by the Gaussian surface is q_a, so the law gives $4\pi r^2 E = q_a/\epsilon_0$ and $E = q_a/4\pi\epsilon_0 r^2$.

(c) For $r > b$ the charge enclosed by the Gaussian surface is $q_a + q_b$, so $4\pi\epsilon_0 E = (q_a + q_b)/\epsilon_0$ and $E = (q_a + q_b)/4\pi\epsilon_0 r^2$.

(d) Consider first a spherical Gaussian with radius just slightly greater than a. The electric field is zero everywhere on this surface, so according to Gauss' law it encloses zero net charge. Since there is no charge in the cavity the charge on the inner surface of the smaller shell is zero. The total charge on the smaller shell is q_a and this must reside on the outer surface. Now consider a spherical Gaussian surface with radius slight larger than the inner radius of the larger shell. This surface also encloses zero net charge, which is the sum of the charge on the outer surface of the smaller shell and the charge on the inner surface of the larger shell. Thus the charge on the inner surface of the larger shell is $-q_a$. The net charge on the larger shell is q_b, with $-q_a$ on its inner surface, so the charge on its outer surface must be $q_b - (-q_a) = q_b + q_a$.

76

(a) The magnitude of the electric field due to a large uniformly charged plate is given by $\sigma/2\epsilon_0$, where σ is the surface charge density. In the region between the oppositely charged plates the fields of the plates are in the same direction, so the net field has magnitude $E = \sigma/\epsilon_0$. The electrical force on an electron has magnitude $eE = e\sigma/\epsilon_0$ and the gravitational force on it is mg, where m is it mass. If these forces are to balance, they must have the same magnitude, so $mg = e\sigma/\epsilon_0$ and

$$\sigma = \frac{mg\epsilon_0}{e} = \frac{(9.11 \times 10^{-31}\,\text{kg})(9.8\,\text{m/s}^2)(8.85 \times 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2)}{1.60 \times 10^{-19}\,\text{C}} = 4.9 \times 10^{-22}\,\text{C/m}^2 .$$

(b) The gravitational force is downward, so the electrical force must be upward. Since an electron is negatively charged the electrical force on it is opposite to the electric field, so the electric field must be downward.

79

(a) Let Q be the net charge on the shell, q_i be the charge on its inner surface and q_o be the charge on its outer surface. Then $Q = q_i + q_o$ and $q_i = Q - q_o = (-10\,\mu\text{C}) - (-14\,\mu\text{C}) = +4\,\mu\text{C}$.

(b) Let q be the charge on the particle. Gauss' law tells us that since the electric field is zero inside the conducting shell the net charge inside any spherical surface that entirely within the shell is zero. Thus the sum of the charge on the particle and on the inner surface of the shell is zero, so $q + q_i = 0$ and $q = -q_i = -4\,\mu\text{C}$.

Chapter 24

3

(a) An ampere is a coulomb per second, so

$$84\,\mathrm{A}\cdot\mathrm{h} = \left(84\,\frac{\mathrm{C}\cdot\mathrm{h}}{\mathrm{s}}\right)\left(3600\,\frac{\mathrm{s}}{\mathrm{h}}\right) = 3.0\times10^{5}\,\mathrm{C}\,.$$

(b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0\times10^{5}\,\mathrm{C})(12\,\mathrm{V}) = 3.6\times10^{6}\,\mathrm{J}$.

5

The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\epsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex\,,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by $\Delta V = E\Delta x = (\sigma/2\epsilon_0)\Delta x$. Thus

$$\Delta x = \frac{2\epsilon_0\,\Delta V}{\sigma} = \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(50\,\mathrm{V})}{0.10\times10^{-6}\,\mathrm{C/m^2}} = 8.8\times10^{-3}\,\mathrm{m}\,.$$

19

(a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\epsilon_0 R$. Thus

$$R = \frac{q}{4\pi\epsilon_0 V} = \frac{(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2})(30\times10^{-12}\,\mathrm{C})}{500\,\mathrm{V}} = 5.4\times10^{-4}\,\mathrm{m}\,.$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus

$$V' = \frac{1}{4\pi\epsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\epsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\,\mathrm{V}) = 790\,\mathrm{V}\,.$$

29

The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk.

Consider a ring of charge with radius r and width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2 + D^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r\,dr}{\sqrt{r^2 + D^2}} = \frac{\sigma r\,dr}{2\epsilon_0\sqrt{r^2 + D^2}}\,.$$

The total potential at P is

$$V = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2 + D^2}} = \frac{\sigma}{2\epsilon_0}\sqrt{r^2 + D^2}\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2 + D^2} - D\right]\,.$$

The potential V_{sq} at P due to a single quadrant is

$$\begin{aligned} V_{\text{sq}} &= \frac{V}{4} = \frac{\sigma}{8\epsilon_0}\left[\sqrt{R^2 + D^2} - D\right] \\ &= \frac{7.73\times 10^{-15}\,\text{C/m}^2}{8(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\,\text{m})^2 + (0.259\,\text{m})^2} - 0.259\,\text{m}\right] \\ &= 4.71\times 10^{-5}\,\text{V}\,. \end{aligned}$$

39

Take the negatives of the partial derivatives of the electric potential with respect to the coordinates and evaluate the results for $x = 3.00$ m, $y = -2.00$ m, and $z = 4.00$ m. This yields

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -(2.00\,\text{V/m}^4)yz^2 = -(2.00\,\text{V/m}^4)((-2.00\,\text{m})(4.00\,\text{m})^2 = 64.0\,\text{V/m}\,, \\ E_y &= -\frac{\partial V}{\partial y} = -(2.00\,\text{V/m}^4)xz^2 = -(2.00\,\text{V/m}^4)(3.00\,\text{m})(4.00\,\text{m})^2 = -96.0\,\text{V/m}\,, \\ E_z &= -\frac{\partial V}{\partial z} = -2(2.00\,\text{V/m}^4)xyz = -2(2.00\,\text{V/m}^4)(3.00\,\text{m})(-2.00\,\text{m})(4.00\,\text{m}) = 96.0\,\text{V/m}\,. \end{aligned}$$

The magnitude of the electric field is

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0\,\text{V/m})^2 + (-96.0\,\text{V/m})^2 + (96.0\,\text{V/m})^2} = 1.50\times 10^2\,\text{V/m}\,.$$

41

The work required is equal to the potential energy of the system, relative to a potential energy of zero for infinite separation. Number the particles 1, 2, 3, and 4, in clockwise order starting with the particle in the upper left corner of the arrangement. The potential energy of the interaction of particles 1 and 2 is

$$\begin{aligned} U_{12} &= \frac{q_1 q_2}{4\pi\epsilon_0 a} = \frac{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.30\times 10^{-12}\,\text{C})(-2.30\times 10^{-12}\,\text{C})}{0.640\,\text{m}} \\ &= -7.43\times 10^{-14}\,\text{J}\,. \end{aligned}$$

The distance between particles 1 and 3 is $\sqrt{2}a$ and both these particles are positively charged, so the potential energy of the interaction between particles 1 and 3 is $U_{13} = -U_{12}/\sqrt{2} = +5.25\,\times$

10^{-14} J. The potential energy of the interaction between particles 1 and 4 is $U_{14} = U_{12} = -7.43 \times 10^{-14}$ J. The potential energy of the interaction between particles 2 and 3 is $U_{23} = U_{12} = -7.43 \times 10^{-14}$ J. The potential energy of the interaction between particles 2 and 4 is $U_{24} = U_{13} = 5.25 \times 10^{-14}$ J. The potential energy of the interaction between particles 3 and 4 is $U_{34} = U_{12} = -7.43 \times 10^{-14}$ J.

The total potential energy of the system is

$$\begin{aligned} U &= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \\ &= -7.43 \times 10^{-14}\,\mathrm{J} + 5.25 \times 10^{-14}\,\mathrm{J} - 7.43 \times 10^{-14}\,\mathrm{J} - 7.43 \times 10^{-14}\,\mathrm{J} \\ &\qquad - 7.43 \times 10^{-14}\,\mathrm{J} + 5.25 \times 10^{-14}\,\mathrm{J} = -1.92 \times 10^{-13}\,\mathrm{J}\,. \end{aligned}$$

This is equal to the work that must be done to assemble the system from infinite separation.

59

(a) Use conservation of mechanical energy. The potential energy when the moving particle is at any coordinate y is qV, where V is the electric potential produced at that place by the two fixed particles. That is,

$$U = q\frac{2Q}{4\pi\epsilon_0\sqrt{x^2+y^2}}\,,$$

where x is the coordinate and Q is the charge of either one of the fixed particles. The factor 2 appears since the two fixed particles produce the same potential at points on the y axis. Conservation of mechanical energy yields

$$K_f = K_i + q\frac{2Q}{4\pi\epsilon_0\sqrt{x^2+y_i^2}} - q\frac{2Q}{4\pi\epsilon_0\sqrt{x^2+y_f^2}} = K_i + \frac{2qQ}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{x^2+y_i^2}} - \frac{1}{\sqrt{x^2+y_f^2}}\right)\,,$$

where K is the kinetic energy of the moving particle, the subscript i refers to the initial position of the moving particle, and the subscript f refers to the final position. Numerically

$$K_f = 1.2\,\mathrm{J} + \frac{2(-15\times10^{-6}\,\mathrm{C})(50\times10^{-6}\,\mathrm{C})}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})}\left[\frac{1}{\sqrt{(3.0\,\mathrm{m})^2+(4.0\,\mathrm{m})^2}} - \frac{1}{\sqrt{(3.0\,\mathrm{m})^2}}\right] = 3.0\,\mathrm{J}\,.$$

(b) Now $K_f = 0$ and we solve the energy conservation equation for y_f. Conservation of energy first yields $U_f = K_i + U_i$. The initial potential energy is

$$U_i = \frac{2qQ}{4\pi\epsilon_0\sqrt{x^2+y_i^2}} = \frac{2(-15\times10^{-6}\,\mathrm{C})(50\times10^{-6}\,\mathrm{C})}{4\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})\sqrt{(3.0\,\mathrm{m})^2+(4.0\,\mathrm{m})^2}} = -2.7\,\mathrm{J}\,.$$

Thus $K_f = 1.2\,\mathrm{J} - 2.7\,\mathrm{J} = -1.5\,\mathrm{J}$.

Now

$$U_f = \frac{2qQ}{4\pi\epsilon_0\sqrt{x^2+y_f^2}}$$

so

$$y = -\sqrt{\left(\frac{2qQ}{4\pi\epsilon_0 U_f}\right)^2 - x^2} = \sqrt{\left(\frac{2(-15\times 10^{-6}\,\mathrm{C})(50\times 10^{-6}\,\mathrm{C})}{4\pi(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(-1.5\,\mathrm{J})}\right)^2 - (3.0\,\mathrm{m})^2} = -8.5\,\mathrm{m}\,.$$

63

If the electric potential is zero at infinity, then the electric potential at the surface of the sphere is given by $V = q/4\pi\epsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\epsilon_0 rV = \frac{(0.15\,\mathrm{m})(1500\,\mathrm{V})}{8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}} = 2.5\times 10^{-8}\,\mathrm{C}\,.$$

65

(a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\epsilon_0 d/2} = \frac{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times 10^{-8}\,\mathrm{C} - 3.0\times 10^{-8}\,\mathrm{C})}{1.0\,\mathrm{m}} = -1.80\times 10^2\,\mathrm{V}\,.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] \\ &= (8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0\times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}} - \frac{3.0\times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}}\right] \\ &= 2.9\times 10^3\,\mathrm{V}\,. \end{aligned}$$

(c) The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] \\ &= (8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0\times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}} - \frac{3.0\times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}}\right] \\ &= -8.9\times 10^3\,\mathrm{V}\,. \end{aligned}$$

75

The initial potential energy of the three-particle system is $U_i = 2(q^2/4\pi\epsilon_0 L) + U_{\text{fixed}}$, where q is the charge on each particle, L is the length of a triangle side, and U_{fixed} is the potential energy associated with the interaction of the two fixed particles. The factor 2 appears since the potential energy is the same for the interaction of the movable particle and each of the fixed particles. The final potential energy is $U_f = 2[q^2/4\pi\epsilon_0(L/2)] + U_{\text{fixed}}$ and the change in the potential energy is

$$\Delta U = U_f - U_i = \frac{2q^2}{4\pi\epsilon_0}\left(\frac{2}{L} - \frac{1}{L}\right) = \frac{2q^2}{4\pi\epsilon_0 L}\,.$$

This is the work that is done by the external agent. If P is the rate with energy is supplied by the agent and t is the time for the move, then $Pt = \Delta U$, and

$$t = \frac{\Delta U}{P} = \frac{2q^2}{4\pi\epsilon_0 LP} = \frac{2(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(0.12\,\mathrm{C})^2}{(1.7\,\mathrm{m})(0.83\times10^3\,\mathrm{W})} = 1.83\times10^5\,\mathrm{s}\,.$$

This is 2.1 d.

77

(a) Use Gauss' law to find an expression for the electric field. The Gaussian surface is a cylindrical surface that is concentric with the cylinder and has a radius r that is greater than the radius of the cylinder. The electric field is normal to the Gaussian surface and has uniform magnitude on it, so the integral in Gauss' law is $\oint \vec{E}\cdot d\vec{A} = 2\pi rEL$, where L is the length of the Gaussian surface. The charge enclosed is λL, where λ is the charge per unit length on the cylinder. Thus $2\pi rRLE = \lambda L/\epsilon_0$ and $E = \lambda/2\pi\epsilon_0 r$.

Let E_B be the magnitude of the field at B and r_B be the distance from the central axis to B. Let E_C be the magnitude of the field at C and r_C be the distance from the central axis to C. Since E is inversely proportional to the distance from the central axis,

$$E_C = \frac{r_B}{r_C}E_B = \frac{2.0\,\mathrm{cm}}{5.0\,\mathrm{cm}}(160\,\mathrm{N/C}) = 64\,\mathrm{N/C}\,.$$

(b) The magnitude of the field a distance r from the central axis is $E = (r_B/r)E_B$, so the potential difference of points B and C is

$$\begin{aligned} V_B - V_C &= -\int_{r_C}^{r_B}\frac{r_B}{r}E_B\,dr = -r_B E_B \ln\left(\frac{r_B}{r_C}\right) \\ &= -(0.020\,\mathrm{m})(160\,\mathrm{N/C})\ln\left(\frac{0.020\,\mathrm{m}}{0.050\,\mathrm{m}}\right) = 2.9\,\mathrm{V}\,. \end{aligned}$$

(c) The cylinder is conducting, so all points inside have the same potential, namely V_B, so $V_A - V_B = 0$.

85

Consider a point on the z axis that has coordinate z. All points on the ring are the same distance from the point. The distance is $r = \sqrt{R^2+z^2}$, where R is the radius of the ring. If the electric potential is taken to be zero at points that are infinitely far from the ring, then the potential at the point is

$$V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+z^2}}\,,$$

where Q is the charge on the ring. Thus

$$\begin{aligned} V_B - V_A &= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{R}\right] \\ &= (8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(16.0\times10^{-6}\,\mathrm{C})\left[\frac{1}{\sqrt{(0.0300\,\mathrm{m})^2+(0.0400\,\mathrm{m})^2}} - 0.300\,\mathrm{m}\right] \\ &= -1.92\times10^6\,\mathrm{V}\,. \end{aligned}$$

93

(a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}\,,$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)}$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right)\,.$$

Gauss' law yields

$$4\pi\epsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right)$$

and the magnitude of the electric field is

$$E = \frac{Q}{4\pi\epsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}\,.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V &= V_s - \int_{r_2}^{r} E\,\mathrm{d}r = V_s - \frac{Q}{4\pi\epsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)\mathrm{d}r \\ &= V_s - \frac{Q}{4\pi\epsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right)\,. \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\epsilon_0 r_2$. Make this substitution and collect like terms to find

$$V = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

Since $\rho = 3Q/4\pi(r_2^3 - r_1^3)$ this can also be written

$$V = \frac{\rho}{3\epsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. Put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\epsilon_0}\frac{3(r_2^2 - r_1^2)}{2(r_2^3 - r_1^3)},$$

or in terms of the charge density

$$V = \frac{\rho}{2\epsilon_0}(r_2^2 - r_1^2).$$

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

95

The electric potential of a dipole at a point a distance r away is given by Eq. 24–30:

$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2},$$

where p is the magnitude of the dipole moment and θ is the angle between the dipole moment and the position vector of the point. The potential at infinity was taken to be zero. Take the z axis to be the dipole axis and consider a point with z positive (on the positive side of the dipole). For this point $r = z$ and $\theta = 0$. The z component of the electric field is

$$E_z = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial z}\left(\frac{p}{4\pi\epsilon_0 z^2}\right) = \frac{p}{2\pi\epsilon_0 z^3}.$$

This is the only nonvanishing component at a point on the dipole axis.

For a point with a negative value of z, $r = -z$ and $\cos\theta = -1$, so

$$E_z = -\frac{\partial}{\partial z}\left(\frac{-p}{4\pi\epsilon_0 z^2}\right) = -\frac{p}{2\pi\epsilon_0 z^3}.$$

103

(a) The electric potential at the surface of the sphere is given by $V = q/4\pi\epsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. The charge on the sphere when the potential reaches 1000 V is

$$q = 4\pi\epsilon_0 rV = \frac{(0.010\,\text{m})(1000\,\text{V})}{8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times 10^{-9}\,\text{C}.$$

The number of electrons that enter the sphere is $N = q/e = (1.11 \times 10^{-9}\,\mathrm{C})/(1.60 \times 10^{-19}\,\mathrm{C}) = 6.95 \times 10^9$. Let R be the decay rate and t be the time for the potential to reach it final value. Since half the resulting electrons enter the sphere $N = (P/2)t$ and $t = 2N/P = 2(6.95 \times 10^9)/(3.70 \times 10^8\,\mathrm{s}^{-1}) = 38\,\mathrm{s}$.

(b) The increase in temperature is $\Delta T = N\Delta E/C$, where E is the energy deposited by a single electron and C is the heat capacity of the sphere. Since $N = (P/2)t$, this is $\Delta T = (P/2)t\Delta E/C$ and

$$t = \frac{2C\,\Delta T}{P\,\Delta E} = \frac{2(14\,\mathrm{J/K})(5.0\,\mathrm{K})}{(3.70 \times 10^8\,\mathrm{s}^{-1})(100 \times 10^3\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})} = 2.4 \times 10^7\,\mathrm{s}\,.$$

This is about 280 d.

Chapter 25

5

(a) The capacitance of a parallel-plate capacitor is given by $C = \epsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus

$$C = \frac{\epsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12}\,\mathrm{F/m})\pi(8.20 \times 10^{-2}\,\mathrm{m})^2}{1.30 \times 10^{-3}\,\mathrm{m}} = 1.44 \times 10^{-10}\,\mathrm{F} = 144\,\mathrm{pF}\,.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus $q = (1.44 \times 10^{-10}\,\mathrm{F})(120\,\mathrm{V}) = 1.73 \times 10^{-8}\,\mathrm{C} = 17.3\,\mathrm{nC}$.

15

The charge initially on the charged capacitor is given by $q = C_1 V_0$, where C_1 (= 100 pF) is the capacitance and V_0 (= 50 V) is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1 V$, where v (= 35 V) is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substitute $C_1 V_0$ for q and $C_1 V$ for q_1 to obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50\,\mathrm{V} - 35\,\mathrm{V}}{35\,\mathrm{V}}(100\,\mathrm{pF}) = 43\,\mathrm{pF}\,.$$

19

(a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\mathrm{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

The equivalent capacitance is $C_{\mathrm{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}\,\mathrm{F}$. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = (1.0 \times 10^{-6}\,\mathrm{F})(100\,\mathrm{V}) = 1.0 \times 10^{-4}\,\mathrm{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = (3.0 \times 10^{-6}\,\mathrm{F})(100\,\mathrm{V}) = 3.0 \times 10^{-4}\,\mathrm{C}\,,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\mathrm{C} - 1.0 \times 10^{-4}\,\mathrm{C} = 2.0 \times 10^{-4}\,\mathrm{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\mathrm{C}}{4.0 \times 10^{-6}\,\mathrm{F}} = 50\,\mathrm{V}\,.$$

(b) The charge on capacitor 1 is now $q_1 = C_1 V_{ab} = (1.0 \times 10^{-6}\,\mathrm{F})(50\,\mathrm{V}) = 5.0 \times 10^{-5}\,\mathrm{C}$.

(c) The charge on capacitor 2 is now $q_2 = C_2 V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

29

The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is $U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2.0 \times 10^{-6}\,\text{F} + 4.0 \times 10^{-6}\,\text{F})(300\,\text{V})^2 = 0.27\,\text{J}$.

35

(a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\epsilon_0 A/d$, the charge is $q = CV = \epsilon_0 AV/d$. After the plates are pulled apart, their separation is d' and the potential difference is V'. Then $q = \epsilon_0 AV'/d'$ and

$$V' = \frac{d'}{\epsilon_0 A}\, q = \frac{d'}{\epsilon_0 A}\frac{\epsilon_0 A}{d}\, V = \frac{d'}{d} V = \frac{8.00\,\text{mm}}{3.00\,\text{mm}}(6.00\,\text{V}) = 16.0\,\text{V}\,.$$

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{\epsilon_0 AV^2}{2d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(6.00\,\text{V})}{2(3.00 \times 10^{-3}\,\text{mm})} = 4.51 \times 10^{-11}\,\text{J}$$

and the final energy stored is

$$U_f = \frac{1}{2}C'(V')^2 = \frac{1}{2}\frac{\epsilon_0 A}{d'}(V')^2 = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(16.0\,\text{V})}{2(8.00 \times 10^{-3}\,\text{mm})} = 1.20 \times 10^{-10}\,\text{J}\,.$$

(c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = 1.20 \times 10^{-10}\,\text{J} - 4.51 \times 10^{-11}\,\text{J} = 7.49 \times 10^{-11}\,\text{J}$.

43

The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)}\,,$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. See Eq. 25–14. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\epsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85 \times 10^{-12}\,\text{F/m})}{\ln\left[(0.60\,\text{mm})/(0.10\,\text{mm})\right]} = 8.1 \times 10^{-11}\,\text{F/m} = 81\,\text{pF/m}\,.$$

45

The capacitance is given by $C = \kappa C_0 = \kappa\epsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus $d = V/E$ and $C = \kappa\epsilon_0 AE/V$. Solve for A:

$$A = \frac{CV}{\kappa\epsilon_0 E}\,.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0 \times 10^{-8}\,\mathrm{F})(4.0 \times 10^{3}\,\mathrm{V})}{2.8(8.85 \times 10^{-12}\,\mathrm{F/m})(18 \times 10^{6}\,\mathrm{V/m})} = 0.63\,\mathrm{m}^2\,.$$

51

(a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\epsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\epsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50\,\mathrm{V})(100 \times 10^{-12}\,\mathrm{F})}{5.4(8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)} = 1.0 \times 10^{4}\,\mathrm{V/m}\,.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\,\mathrm{F})(50\,\mathrm{V}) = 5.0 \times 10^{-9}\,\mathrm{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\epsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A}\,,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so the fields tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 AE \\ &= 5.0 \times 10^{-9}\,\mathrm{C} - (8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)(1.0 \times 10^{4}\,\mathrm{V/m}) \\ &= 4.1 \times 10^{-9}\,\mathrm{C} = 4.1\,\mathrm{nC}\,. \end{aligned}$$

61

Capacitors 3 and 4 are in parallel and may be replaced by a capacitor with capacitance $C_{34} = C_3 + C_4 = 30\,\mu\mathrm{F}$. Capacitors 1, 2, and the equivalent capacitor that replaced 3 and 4 are all in series, so the sum of their potential differences must equal the potential difference across the battery. Since all of these capacitors have the same capacitance the potential difference across each of them is one-third the battery potential difference or 3.0 V. The potential difference across capacitor 4 is the same as the potential difference across the equivalent capacitor that replaced 3 and 4, so the charge on capacitor 4 is $q_4 = C_4V_4 = (15 \times 10^{-6}\,\mathrm{F})(3.0\,\mathrm{V}) = 45 \times 10^{-6}\,\mathrm{C}$.

69

(a) and (b) The capacitors have the same plate separation d and the same potential difference V across their plates, so the electric field are the same within them. The magnitude of the field in either one is $E = V/d = (600\,\text{V})/(3.00 \times 10^{-3}\,\text{m}) = 2.00 \times 10^5\,\text{V/m}$.

(c) Let A be the area of a plate. Then the surface charge density on the positive plate is $\sigma_A = q_A/A = C_A V/A = (\epsilon_0 A/d)V/A = \epsilon_0 V/d = \epsilon_0 E = (8.85 \times 10^{-12}\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^5\,\text{V/m}) = 1.77 \times 10^{-6}\,\text{C/m}^2$, where CV was substituted for q and the expression $\epsilon_0 A/d$ for the capacitance of a parallel-plate capacitor was substituted for C.

(d) Now the capacitance is $\kappa\epsilon_0 A/d$, where κ is the dielectric constant. The surface charge density on the positive plate is $\sigma_B = \kappa\epsilon_0 E = \kappa\sigma_A = (2.60)(1.77 \times 10^{-6}\,\text{C/m}^2) = 4.60 \times 10^{-6}\,\text{C/m}^2$.

(e) The electric field in B is produced by the charge on the plates and the induced charge together while the field in A is produced by the charge on the plates alone. since the fields are the same $\sigma_B + \sigma_{\text{induced}} = \sigma_A$, so $\sigma_{\text{induced}} = \sigma_A - \sigma_B = 1.77 \times 10^{-6}\,\text{C/m}^2 - 4.60 \times 10^{-6}\,\text{C/m}^2 = -2.83 \times 10^{-6}\,\text{C/m}^2$.

73

The electric field in the lower region is due to the charge on both plates and the charge induced on the upper and lower surfaces of the dielectric in the region. The charge induced on the dielectric surfaces of the upper region has the same magnitude but opposite sign on the two surfaces and so produces a net field of zero in the lower region. Similarly, the electric field in the upper region is due to the charge on the plates and the charge induced on the upper and lower surfaces of dielectric in that region. Thus the electric field in the upper region has magnitude $E_{\text{upper}} = q\kappa_{\text{upper}}\epsilon_0 A$ and the potential difference across that region is $V_{\text{upper}} = E_{\text{upper}}d$, where d is the thickness of the region. The electric field in the lower region is $E_{\text{lower}} = q\kappa_{\text{lower}}\epsilon_0 A$ and the potential difference across that region is $V_{\text{lower}} = E_{\text{lower}}d$. The sum of the potential differences must equal the potential difference V across the entire capacitor, so

$$V = E_{\text{upper}}d + E_{\text{lower}}d = \frac{qd}{\epsilon_0 A}\left[\frac{1}{\kappa_{\text{upper}}} + \frac{1}{\kappa_{\text{lower}}}\right] .$$

The solution for q is

$$\begin{aligned} q &= \frac{\kappa_{\text{upper}}\kappa_{\text{lower}}}{\kappa_{\text{upper}} + \kappa_{\text{lower}}}\frac{\epsilon_0 A}{d}V = \frac{(3.00)(4.00)}{3.00 + 4.00}\,\frac{(8.85 \times 10^{-12}\,\text{N}\cdot m^2/\text{C}^2)(2.00 \times 10^{-2}\,\text{m}^2)}{2.00 \times 10^{-3}\,\text{m}}(7.00\,\text{V}) \\ &= 1.06 \times 10^{-9}\,\text{C} . \end{aligned}$$

Chapter 26

7

(a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8\,\text{cm}^{-3} = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus

$$J = (2 \times 10^{14}\,\text{m}^{-3})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = 6.4\,\text{A/m}^2\,.$$

(b) Since the particles are positively charged, the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

17

The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is the cross-sectional area of the wire. The cross-sectional area is $A = \pi r^2 = \pi(0.50 \times 10^{-3}\,\text{m})^2 = 7.85 \times 10^{-7}\,\text{m}^2$. Here $r = 0.50\,\text{mm} = 0.50 \times 10^{-3}\,\text{m}$ is the radius of the wire. Thus

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\text{m}^2)}{2.0\,\text{m}} = 2.0 \times 10^{-8}\,\Omega \cdot \text{m}\,.$$

19

The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, $L = (250)2\pi r = (250)(2\pi)(0.12\,\text{m}) = 188.5\,\text{m}$. If r_w is the radius of the wire, its cross-sectional area is $A = \pi r_w^2 = \pi(0.65\times10^{-3}\,\text{m})^2 = 1.33\times10^{-6}\,\text{m}^2$. According to Table 26–1, the resistivity of copper is $1.69 \times 10^{-8}\,\Omega \cdot \text{m}$. Thus

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(188.5\,\text{m})}{1.33 \times 10^{-6}\,\text{m}^2} = 2.4\,\Omega\,.$$

21

Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0\,,$$

where R_0 is the original resistance. Thus $R = 9(6.0\,\Omega) = 54\,\Omega$.

23

The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside radius of conductor B and r_i is its inside radius, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$ and its resistance is

$$R_B = \frac{\rho L}{\pi(r_o^2 - r_i^2)}.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0\,.$$

39

(a) Electrical energy is transferred to internal energy at a rate given by

$$P = \frac{V^2}{R},$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus

$$P = \frac{(120\,\text{V})^2}{14\,\Omega} = 1.0 \times 10^3\,\text{W} = 1.0\,\text{kW}\,.$$

(b) The cost is given by

$$C = (1.0\,\text{kW})(5.0\,\text{h})(\$0.050\,/\text{kW}\cdot\text{h}) = \$0.25\,.$$

43

(a) Let P be the rate of energy dissipation, i be the current in the heater, and V be the potential difference across the heater. They are related by $P = iV$. Solve for i:

$$i = \frac{P}{V} = \frac{1250\,\text{W}}{115\,\text{V}} = 10.9\,\text{A}\,.$$

(b) According to the definition of resistance $V = iR$, where R is the resistance of the heater. Solve for R:

$$R = \frac{V}{i} = \frac{115\,\text{V}}{10.9\,\text{A}} = 10.6\,\Omega\,.$$

(c) The thermal energy E produced by the heater in time t (= 1.0 h = 3600 s) is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.5 \times 10^6\,\text{J}\,.$$

53

(a) and (b) Calculate the electrical resistances of the wires. Let ρ_C be the resistivity of wire C, r_C be its radius, and L_C be its length. Then the resistance of this wire is

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0 \times 10^{-6}\,\Omega\cdot\text{m})\frac{1.0\,\text{m}}{\pi(0.50 \times 10^{-3}\,\text{m})^2} = 2.54\,\Omega\,.$$

Let ρ_D be the resistivity of wire D, r_D be its radius, and L_D be its length. Then the resistance of this wire is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0 \times 10^{-6}\,\Omega\cdot\text{m})\frac{1.0\,\text{m}}{\pi(0.25 \times 10^{-3}\,\text{m})^2} = 5.09\,\Omega\,.$$

If i is the current in the wire, the potential difference between points 1 and 2 is

$$\Delta V_{12} = iR_C = (2.0\,\text{A})(2.54\,\Omega) = 5.1\,\text{V}$$

and the potential difference between points 2 and 3 is

$$\Delta V_{23} = iR_D = (2.0\,\text{A})(5.09\,\Omega) = 10\,\text{V}\,.$$

(c) and (d) The rate of energy dissipation between points 1 and 2 is

$$P_{12} = i^2R_C = (2.0\,\text{A})^2(2.54\,\Omega) = 10\,\text{W}$$

and the rate of energy dissipation between points 2 and 3 is

$$P_{23} = i^2R_D = (2.0\,\text{A})^2(5.09\,\Omega) = 20\,\text{W}\,.$$

55

(a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\,\Delta t}{2e} = \frac{(0.25 \times 10^{-6}\,\text{A})(3.0\,\text{s})}{2(1.6 \times 10^{-19}\,\text{C})} = 2.3 \times 10^{12}\,.$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is, $i = 2eN/t = 2eNv/L$. Thus, $N = iL/2ev$.

Now find the particle speed. The kinetic energy of a particle is

$$K = 20\,\text{MeV} = (20 \times 10^6\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV}) = 3.2 \times 10^{-12}\,\text{J}\,.$$

Since $K = \frac{1}{2}mv^2$, $v = \sqrt{2K/m}$. The mass of an alpha particle is four times the mass of a proton or $m = 4(1.67 \times 10^{-27}\,\text{kg}) = 6.68 \times 10^{-27}\,\text{kg}$, so

$$v = \sqrt{\frac{2(3.2 \times 10^{-12}\,\text{J})}{6.68 \times 10^{-27}\,\text{kg}}} = 3.1 \times 10^7\,\text{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25 \times 10^{-6}\,\text{A})(20 \times 10^{-2}\,\text{m})}{2(1.60 \times 10^{-19}\,\text{C})(3.1 \times 10^{7}\,\text{m/s})} = 5.0 \times 10^{3}\,.$$

(c) Use conservation of energy. The initial kinetic energy is zero, the final kinetic energy is $20\,\text{MeV} = 3.2 \times 10^{-12}\,\text{J}$, the initial potential energy is $qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Conservation of energy leads to $K_f = U_i = 2eV$, so

$$V = \frac{K_f}{2e} = \frac{3.2 \times 10^{-12}\,\text{J}}{2(1.60 \times 10^{-19}\,\text{C})} = 10 \times 10^{6}\,\text{V}\,.$$

59

Let R_H be the resistance at the higher temperature (800° C) and let R_L be the resistance at the lower temperature (200° C). Since the potential difference is the same for the two temperatures, the rate of energy dissipation at the lower temperature is $P_L = V^2/R_L$, and the rate of energy dissipation at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now $R_L = R_H + \alpha R_H\,\Delta T$, where ΔT is the temperature difference $T_L - T_H = -600°$ C. Thus,

$$P_L = \frac{R_H}{R_H + \alpha R_H\,\Delta T}\,P_H = \frac{P_H}{1 + \alpha\,\Delta T} = \frac{500\,\text{W}}{1 + (4.0 \times 10^{-4}\,/°\text{C})(-600°\,\text{C})} = 660\,\text{W}\,.$$

75

If the resistivity is ρ_0 at temperature T_0, then the resistivity at temperature T is $\rho = \rho_0 + \alpha\rho_0(T - T_0)$, where α is the temperature coefficient of resistivity. The solution for T is

$$T = \frac{\rho - \rho_0 + \alpha\rho_0 T_0}{\alpha\rho_0}\,.$$

Substitute $\rho = 2\rho_0$ to obtain

$$T = T_0 + \frac{1}{\alpha} = 20.0°\text{C} + \frac{1}{4.3 \times 10^{-3}\,\text{K}^{-1}} = 250°\text{C}\,.$$

The value of α was obtained from Table 26–1.

Chapter 27

7

(a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. Use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. Solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\text{V} - 6.0\,\text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\,\text{A}\,.$$

A positive value was obtained, so the current is counterclockwise around the circuit.

(b) and (c) If i is the current in a resistor with resistance R, then the power dissipated by that resistor is given by $P = i^2 R$. For R_1 the power dissipated is

$$P_1 = (0.50\,\text{A})^2(4.0\,\Omega) = 1.0\,\text{W}$$

and for R_2 the power dissipated is

$$P_2 = (0.50\,\text{A})^2(8.0\,\Omega) = 2.0\,\text{W}\,.$$

(d) and (e) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For battery 1 the power is

$$P_1 = (0.50\,\text{A})(12\,\text{V}) = 6.0\,\text{W}$$

and for battery 2 it is

$$P_2 = (0.50\,\text{A})(6.0\,\text{V}) = 3.0\,\text{W}\,.$$

(f) and (g) In battery 1, the current is in the same direction as the emf so this battery supplies energy to the circuit. The battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

13

(a) If i is the current and ΔV is the potential difference, then the power absorbed is given by $P = i\,\Delta V$. Thus

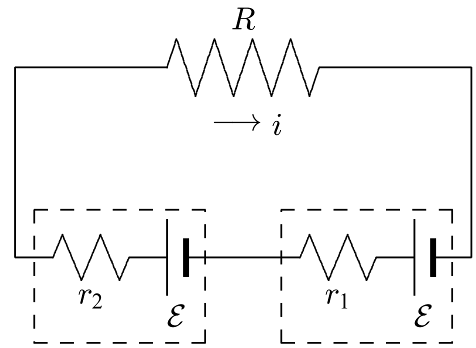
$$\Delta V = \frac{P}{i} = \frac{50\,\text{W}}{1.0\,\text{A}} = 50\,\text{V}\,.$$

Since energy is absorbed, point A is at a higher potential than point B; that is, $V_A - V_B = 50\,\text{V}$.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \mathcal{E}$, where $\mathcal{E}$ is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus $\mathcal{E} = V_A - V_B - iR = 50\,\text{V} - (1.0\,\text{A})(2.0\,\Omega) = 48\,\text{V}$.

(c) A positive value was obtained for $\mathcal{E}$, so it is toward the left. The negative terminal is at B.

21



(a) and (b) The circuit is shown in the diagram to the right. The current is taken to be positive if it is clockwise. The potential difference across battery 1 is given by $V_1 = \mathcal{E} - ir_1$ and for this to be zero, the current must be $i = \mathcal{E}/r_1$. Kirchhoff's loop rule gives $2\mathcal{E} - ir_1 - ir_2 - iR = 0$. Substitute $i = \mathcal{E}/r_1$ and solve for R. You should get $R = r_1 - r_2 = 0.016\,\Omega - 0.012\,\Omega = 0.004\,\Omega$.

Now assume that the potential difference across battery 2 is zero and carry out the same analysis. You should find $R = r_2 - r_1$. Since $r_1 > r_2$ and R must be positive, this situation is not possible. Only the potential difference across the battery with the larger internal resistance can be made to vanish with the proper choice of R.

29

Let r be the resistance of each of the thin wires. Since they are in parallel, the resistance R of the composite can be determined from

$$\frac{1}{R} = \frac{9}{r}\,,$$

or $R = r/9$. Now

$$r = \frac{4\rho\ell}{\pi d^2}$$

and

$$R = \frac{4\rho\ell}{\pi D^2}\,,$$

where ρ is the resistivity of copper. Here $\pi d^2/4$ was used for the cross-sectional area of any one of the original wires and $\pi D^2/4$ was used for the cross-sectional area of the replacement wire. Here d and D are diameters. Since the replacement wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}\,.$$

Solve for D and obtain $D = 3d$.

33

Replace the two resistors on the left with their equivalent resistor. They are in parallel, so the equivalent resistance is $R_{eq} = 1.0\,\Omega$. The circuit now consists of the two emf devices and four resistors. Take the current to be upward in the right-hand emf device. Then the loop rule gives $\mathcal{E}_2 - iR_{eq} - 3iR - \mathcal{E}_2$, where $R = 2.0\,\Omega$. The current is

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_{eq} + 3R} = \frac{12\,\text{V} - 5.0\,\text{V}}{1.0\,\Omega + 3(2.0\,\Omega)} = 1.0\,\text{A}\,.$$

To find the potential at point 1 take a path from ground, through the equivalent resistor and $\mathcal{E}_2$, to the point. The result is $V_1 = iR_{\text{eq}} - \mathcal{E}_1 = (1.0\,\text{A})(1.0\,\Omega) - 12\,\text{V} = -11\,\text{V}$. To find the potential at point 2 continue the path through the lowest resistor on the digram. It is $V_2 = V_1 + iR = -11\,\text{V} + (1.0\,\text{A})(2.0\,\Omega) = -9.0\,\text{V}$.

47

(a) and (b) The copper wire and the aluminum jacket are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is

$$R_C = \frac{\rho_C L}{\pi a^2}$$

and the resistance of the aluminum jacket is

$$R_A = \frac{\rho_A L}{\pi(b^2 - a^2)}\,.$$

Substitute these expressions into $i_C R_C = i_A R_A$ and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}\,.$$

Solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. You should get

$$i_C = \frac{a^2 \rho_C i}{(b^2 - a^2)\rho_C + a^2 \rho_A}$$

and

$$i_A = \frac{(b^2 - a^2)\rho_C i}{(b^2 - a^2)\rho_C + a^2 \rho_A}\,.$$

The denominators are the same and each has the value

$$\begin{aligned}(b^2 - a^2)\rho_C + a^2\rho_A &= \left[(0.380 \times 10^{-3}\,\text{m})^2 - (0.250 \times 10^{-3}\,\text{m})^2\right](1.69 \times 10^{-8}\,\Omega\cdot\text{m}) \\ &\qquad + (0.250 \times 10^{-3}\,\text{m})^2(2.75 \times 10^{-8}\,\Omega\cdot\text{m}) \\ &= 3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3\,.\end{aligned}$$

Thus

$$i_C = \frac{(0.250 \times 10^{-3}\,\text{m})^2(2.75 \times 10^{-8}\Omega\cdot\text{m})(2.00\,\text{A})}{3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3} = 1.11\,\text{A}$$

and

$$\begin{aligned}i_A &= \frac{\left[(0.380 \times 10^{-3}\,\text{m})^2 - (0.250 \times 10^{-3}\,\text{m})^2\right](1.69 \times 10^{-8}\,\Omega\cdot\text{m})(2.00\,\text{A})}{3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3} \\ &= 0.893\,\text{A}\,.\end{aligned}$$

(c) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{\pi(0.250 \times 10^{-3}\,\mathrm{m})^2(12.0\,\mathrm{V})}{(1.11\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega\cdot\mathrm{m})} = 126\,\mathrm{m}\,.$$

57

During charging the charge on the positive plate of the capacitor is given by Eq. 27–33, with $RC = \tau$. That is,

$$q = C\mathcal{E}\left[1 - e^{-t/\tau}\right],$$

where C is the capacitance, $\mathcal{E}$ is applied emf, and τ is the time constant. You want the time for which $q = 0.990C\mathcal{E}$, so

$$0.990 = 1 - e^{-t/\tau}\,.$$

Thus

$$e^{-t/\tau} = 0.010\,.$$

Take the natural logarithm of both sides to obtain $t/\tau = -\ln 0.010 = 4.61$ and $t = 4.61\tau$.

65

(a), (b), and (c) At $t = 0$, the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0\,,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0\,.$$

Since the resistances are all the same, you can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\mathrm{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\mathrm{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\mathrm{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\mathrm{A}\,.$$

(d), (e), and (f) At $t = \infty$, the capacitor is fully charged and the current in the capacitor branch is zero. Then $i_1 = i_2$ and the loop rule yields

$$\mathcal{E} - i_1 R_1 - i_1 R_2 = 0\,.$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\ \text{V}}{2(0.73 \times 10^6\ \Omega)} = 8.2 \times 10^{-4}\ \text{A}\,.$$

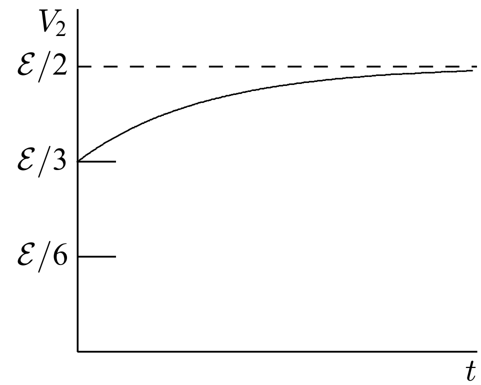
(g) and (h) The potential difference across resistor 2 is $V_2 = i_2 R_2$. At $t = 0$ it is

$$V_2 = (5.5 \times 10^{-4}\ \text{A})(0.73 \times 10^6\ \Omega) = 4.0 \times 10^2\ \text{V}$$

and at $t = \infty$ it is

$$V_2 = (8.2 \times 10^{-4}\ \text{A})(0.73 \times 10^6\ \Omega) = 6.0 \times 10^2\ \text{V}\,.$$

(i) The graph of V_2 versus t is shown to the right.



73

As the capacitor discharges the potential difference across its plates at time t is given by $V = V_0 e^{-t/\tau}$, where V_0 is the potential difference at time $t = 0$ and τ is the capacitive time constant. This equation is solved for the time constant, with result

$$\tau = -\frac{t}{\ln(V/V_0)}\,.$$

Since the time constant is $\tau = RC$, where RR is the resistance and C is the capacitance,

$$R = -\frac{t}{C\ln(V/V_0)}\,.$$

For the smaller time interval

$$R = -\frac{10.0 \times 10^{-6}\ \text{s}}{(0.220 \times 10^{-6}\ \text{F})\ln\left(\dfrac{0.800\ \text{V}}{5.00\ \text{V}}\right)} = 24.8\ \Omega\,.$$

and for the larger time interval

$$R = -\frac{6.00 \times 10^{-3}\ \text{s}}{(0.220 \times 10^{-6}\ \text{F})\ln\left(\dfrac{0.800\ \text{V}}{5.00\ \text{V}}\right)} = 1.49 \times 10^4\ \Omega\,.$$

75

(a) Let i be the current, which is the same in both wires, and $\mathcal{E}$ be the applied potential difference. Then the loop equation gives $\mathcal{E} - iR_A - iR_B = 0$ and the current is

$$i = \frac{\mathcal{E}}{R_A + R_B} = \frac{60.0\ \text{V}}{0.127\ \Omega + 0.729\ \Omega} = 70.1\ \text{A}\,.$$

The current density in wire A is

$$J_A = \frac{i}{\pi r_A^2} = \frac{70.1\,\text{A}}{\pi(1.30 \times 10^{-3}\,\text{m})^2} = 1.32 \times 10^7\,\text{A/m}^2\,.$$

(b) The potential difference across wire A is $V_A = iR_A = (70.1\,\text{A})(0.127\,\Omega) = 8.90\,\text{V}$.

(c) The resistance is $R_A = \rho_A L/A$, where ρ is the resistivity, A is the cross-sectional area, and L is the length. The resistivity of wire A is

$$\rho_A = \frac{R_A A}{L} = \frac{(0.127\,\Omega)\pi(1.30 \times 10^{-3}\,\text{m})^2}{40.0\,\text{m}} = 1.69 \times 10^{-8}\,\Omega\cdot\text{m}\,.$$

According to Table 26–1 the material is copper.

(d) Since wire B has the same diameter and length as wire A and carries the same current, the current density in it is the same, $1.32 \times 10^7\,\text{A/m}^2$.

(e) The potential difference across wire B is $V_B = iR_B = (70.1\,\text{A})(0.729\,\Omega) = 51.1\,\text{V}$.

(f) The resistivity of wire B is

$$\rho_B = \frac{R_B A}{L} = \frac{(0.729\,\Omega)\pi(1.30 \times 10^{-3}\,\text{m})^2}{40.0\,\text{m}} = 9.68 \times 10^{-8}\,\Omega\cdot\text{m}\,.$$

According to Table 26–1 the material is iron.

77

The three circuit elements are in series, so the current is the same in all of them. Since the battery is discharging, the potential difference across its terminals is $V_{\text{batt}} = \mathcal{E} - ir$, where $\mathcal{E}$ is its emf and r is its internal resistance. Thus

$$r = \frac{\mathcal{E} - V}{i} = \frac{12\,\text{V} - 11.4\,\text{V}}{50\,\text{A}} = 0.012\,\Omega\,.$$

This is less than $0.0200\,\Omega$, so the battery is not defective.

The resistance of the cable is $R_{\text{cable}} = V_{\text{cable}}/i = (3.0\,\text{V})/(50\,\text{A} = 0.060\,\Omega$, which is greater than $0.040\,\Omega$. The cable is defective.

The potential difference across the motor is $V_{\text{motor}} = 11.4\,\text{V} - 3.0\,\text{V} = 8.4\,\text{V}$ and its resistance is $R_{\text{motor}} = V_{\text{motor}}/i = (8.4\,\text{V})/(50\,\text{A}) = 0.17\,\Omega$, which is less than $0.200\,\Omega$. The motor is not defective.

85

Let R_{S0} be the resistance of the silicon resistor at 20° and R_{I0} be the resistance of the iron resistor at that temperature. At some other temperature T the resistance of the silicon resistor is $R_S = R_{S0} + \alpha_S R_{S0}(T - 20°\text{C})$ and the resistance of the iron resistor is $R_I = R_{I0} + \alpha_I R_{I0}(T - 20°\text{C})$. Here α_S and α_I are the temperature coefficients of resistivity. The resistors are series so the resistance of the combination is

$$R = R_{S0} + R_{I0} + (\alpha_S R_{S0} + \alpha_I R_{I0})(T - 20°\text{C})\,.$$

We want $R_{S0} + R_{I0}$ to be $1000\,\Omega$ and $\alpha_S R_{S0} + \alpha_I R_{I0}$ to be zero. Then the resistance of the combination will be independent of the temperature.

The second equation gives $R_{I0} = -(\alpha_S/\alpha_I)R_{S0}$ and when this is used to substitute for R_{I0} in the first equation the result is $R_{S0} - (\alpha_S/\alpha_I)R_{S0} = 1000\,\Omega$. The solution for R_{S0} is

$$R_{S0} = \frac{1000\,\Omega}{\dfrac{\alpha_S}{\alpha_I} - 1} = \frac{1000\,\Omega}{\dfrac{-70 \times 10^{-3}\,\mathrm{K}^{-1}}{6.5 \times 10^{-3}\,\mathrm{K}^{-1}} - 1} = 85\,\Omega\,,$$

where values for the temperature coefficients of resistivity were obtained from Table 26–1. The resistance of the iron resistor is $R_{I0} = 1000\,\Omega - 85\,\Omega = 915\,\Omega$.

95

When the capacitor is fully charged the potential difference across its plates is $\mathcal{E}$ and the energy stored in it is $U = \frac{1}{2}C\mathcal{E}^2$.

(a) The current is given as a function of time by $i = (\mathcal{E}/R)e^{-t/\tau}$, where τ (= RC) is the capacitive time constant. The rate with which the emf device supplies energy is $P_{\mathcal{E}} = i\mathcal{E}$ and the energy supplied in fully charging the capacitor is

$$E_{\mathcal{E}} = \int_0^\infty P_{\mathcal{E}}\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/\tau}\,dt = \frac{\mathcal{E}^2\tau}{R} = \frac{\mathcal{E}^2 RC}{R} = C\mathcal{E}^2\,.$$

This is twice the energy stored in the capacitor.

(b) The rate with which energy is dissipated in the resistor is $P_R = i^2R$ and the energy dissipated as the capacitor is fully charged is

$$E_R = \int_0^\infty P_R\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/\tau}\,dt = \frac{\mathcal{E}^2\tau}{2R} = \frac{\mathcal{E}^2 RC}{2R} = \tfrac{1}{2}C\mathcal{E}^2\,.$$

97

(a) Immediately after the switch is closed the capacitor is uncharged and since the charge on the capacitor is given by $q = CV_C$, the potential difference across its plates is zero. Apply the loop rule to the right-hand loop to find that the potential difference across R_2 must also be zero. Now apply the loop rule to the left-hand loop to find that $\mathcal{E} - i_1R_1 = 0$ and $i_1 = \mathcal{E}/R_1 = (30\,\mathrm{V})/(20 \times 10^3\,\Omega) = 1.5 \times 10^{-3}\,\mathrm{A}$.

(b) Since the potential difference across R_2 is zero and this potential difference is given by $V_{R2} = i_2R_2$, $i_2 = 0$.

(c) A long time later, when the capacitor is fully charged, the current is zero in the capacitor branch and the current is the same in the two resistors. The loop rule applied to the left-hand loop gives $\mathcal{E} - iR_1 - iR_2 = 0$, so $i = \mathcal{E}/(R_1 + R_2) = (30\,\mathrm{V})/(20 \times 10^3\,\Omega + 10 \times 10^3\,\Omega) = 1.0 \times 10^{-3}\,\mathrm{A}$.

99

(a) R_2 and R_3 are in parallel, with an equivalent resistance of $R_2R_3/(R_2 + R_3)$, and this combination is in series with R_1, so the circuit can be reduced to a single loop with an emf $\mathcal{E}$ and a resistance $R_{\mathrm{eq}} = R_1 + R_2R_3/(R_2 + R_3) = (R_1R_2 + R_1R_3 + R_2R_3)/(R_1 + R_2)$. The current is

$$i = \frac{\mathcal{E}}{R_{\mathrm{eq}}} = \frac{(R_2 + R_3)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}\,.$$

The rate with which the battery supplies energy is

$$P = i\mathcal{E} = \frac{(R_2 + R_3)\mathcal{E}^2}{R_1R_2 + R_1R_3 + R_2R_3}\,.$$

The derivative with respect to R_3 is

$$\frac{dP}{dR_3} = \frac{\mathcal{E}^2}{R_1R_2 + R_1R_3 + R_2R_3} - \frac{(R_2 + R_3)(R_1 + R_2)\mathcal{E}^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2} = -\frac{\mathcal{E}^2R_2^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}\,,$$

where the last form was obtained with a little algebra. The derivative is negative for all (positive) values of the resistances, so P has its maximum value for $R_3 = 0$.

(b) Substitute $R_3 = 0$ in the expression for P to obtain

$$P = \frac{R_1\mathcal{E}^2}{R_1R_2} = \frac{\mathcal{E}^2}{R_1} = \frac{12.0\,\mathrm{V}}{10.0\,\Omega} = 14.4\,\mathrm{W}\,.$$

101

If the batteries are connected in series the total emf in the circuit is $N\mathcal{E}$ and the equivalent resistance is $R + nr$, so the current is $i = N\mathcal{E}/(R + Nr)$. If $R = r$, then $i = N\mathcal{E}/(N + 1)r$.

If the batteries are connected in parallel then the emf in the circuit is $\mathcal{E}$ and the equivalent resistance is $R + r/N$, so the current is $i = \mathcal{E}/(R + r/N) = N\mathcal{E}/(NR + r)$. If $R = r$, $i = N\mathcal{E}/(N + 1)r$, the same as when they are connected in series.

Chapter 28

3

(a) The magnitude of the magnetic force on the proton is given by $F_B = evB\sin\phi$, where v is the speed of the proton, B is the magnitude of the magnetic field, and ϕ is the angle between the particle velocity and the field when they are drawn with their tails at the same point. Thus

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\mathrm{N}}{(1.60\times10^{-19}\,\mathrm{C})(2.60\times10^{-3}\,\mathrm{T})\sin 23.0^\circ} = 4.00\times10^{5}\,\mathrm{m/s}\,.$$

(b) The kinetic energy of the proton is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67\times10^{-27}\,\mathrm{kg})(4.00\times10^{5}\,\mathrm{m/s})^2 = 1.34\times10^{-16}\,\mathrm{J}\,.$$

This is $(1.34\times10^{-16}\,\mathrm{J})/(1.60\times10^{-19}\,\mathrm{J/eV}) = 835\,\mathrm{eV}$.

17

(a) Since the kinetic energy is given by $K = \frac{1}{2}mv^2$, where m is the mass of the electron and v is its speed,

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.20\times10^{3}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{9.11\times10^{-31}\,\mathrm{kg}}} = 2.05\times10^{7}\,\mathrm{m/s}\,.$$

(b) The magnitude of the magnetic force is given by evB and the acceleration of the electron is given by v^2/r, where r is the radius of the orbit. Newton's second law is $evB = mv^2/r$, so

$$B = \frac{mv}{er} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(2.05\times10^{7}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(25.0\times10^{-2}\,\mathrm{m})} = 4.68\times10^{-4}\,\mathrm{T} = 468\,\mu\mathrm{T}\,.$$

(c) The frequency f is the number of times the electron goes around per unit time, so

$$f = \frac{v}{2\pi r} = \frac{2.05\times10^{7}\,\mathrm{m/s}}{2\pi(25.0\times10^{-2}\,\mathrm{m})} = 1.31\times10^{7}\,\mathrm{Hz} = 13.1\,\mathrm{MHz}\,.$$

(d) The period is the reciprocal of the frequency:

$$T = \frac{1}{f} = \frac{1}{1.31\times10^{7}\,\mathrm{Hz}} = 7.63\times10^{-8}\,\mathrm{s} = 76.3\,\mathrm{ns}\,.$$

29

(a) If v is the speed of the positron, then $v\sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv\sin\phi = m(v\sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (mv/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m}{eB} = \frac{2\pi(9.11\times10^{-31}\,\mathrm{kg})}{(1.60\times10^{-19}\,\mathrm{C})(0.100\,\mathrm{T})} = 3.58\times10^{-10}\,\mathrm{s}\,.$$

The expression for r was substituted to obtain the second expression for T.

(b) The pitch p is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. Use the kinetic energy to find the speed: $K = \frac{1}{2}mv^2$ yields

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.0 \times 10^3\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 2.651 \times 10^7\,\mathrm{m/s}\,.$$

Thus

$$p = (2.651 \times 10^7\,\mathrm{m/s})(3.58 \times 10^{-10}\,\mathrm{s})\cos 89.0° = 1.66 \times 10^{-4}\,\mathrm{m}\,.$$

(c) The orbit radius is

$$r = \frac{mv\sin\phi}{eB} = \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(2.651 \times 10^7\,\mathrm{m/s})\sin 89.0°}{(1.60 \times 10^{-19}\,\mathrm{C})(0.100\,\mathrm{T})} = 1.51 \times 10^{-3}\,\mathrm{m}\,.$$

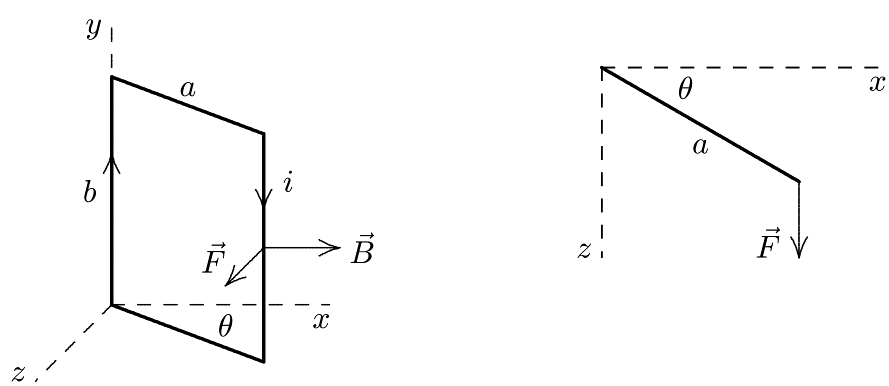
41

(a) The magnitude of the magnetic force on the wire is given by $F_B = iLB\sin\phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus

$$F_B = (5000\,\mathrm{A})(100\,\mathrm{m})(60.0 \times 10^{-6}\,\mathrm{T})\sin 70° = 28.2\,\mathrm{N}\,.$$

(b) Apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

47



The situation is shown in the left diagram above. The y axis is along the hinge and the magnetic field is in the positive x direction. A torque around the hinge is associated with the wire opposite the hinge and not with the other wires. The force on this wire is in the positive z direction and has magnitude $F = NibB$, where N is the number of turns.

The right diagram shows the view from above. The magnitude of the torque is given by

$$\begin{aligned}\tau &= Fa\cos\theta = NibBa\cos\theta \\ &= 20(0.10\,\mathrm{A})(0.10\,\mathrm{m})(0.50 \times 10^{-3}\,\mathrm{T})(0.050\,\mathrm{m})\cos 30° \\ &= 4.3 \times 10^{-3}\,\mathrm{N\cdot m}\,.\end{aligned}$$

Use the right-hand rule to show that the torque is directed downward, in the negative y direction. Thus $\vec{\tau} = -(4.3 \times 10^{-3}\,\mathrm{N \cdot m})\hat{\mathrm{j}}$.

55

(a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\mathrm{A \cdot m^2}}{(160)(\pi)(0.0190\,\mathrm{m})^2} = 12.7\,\mathrm{A}\,.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by $\tau = \mu B = (2.30\,\mathrm{A \cdot m^2})(35.0 \times 10^{-3}\,\mathrm{T}) = 8.05 \times 10^{-2}\,\mathrm{N \cdot m}$.

59

The magnitude of a magnetic dipole moment of a current loop is given by $\mu = iA$, where i is the current in the loop and A is the area of the loop. Each of these loops is a circle and its area is given by $A = \pi R^2$, where R is the radius. Thus the dipole moment of the inner loop has a magnitude of $\mu_i = i\pi r_1^2 = (7.00\,\mathrm{A})\pi(0.200\,\mathrm{m})^2 = 0.880\,\mathrm{A \cdot m^2}$ and the dipole moment of the outer loop has a magnitude of $\mu_o = i\pi r_2^2 = (7.00\,\mathrm{A}\pi(0.300\,\mathrm{m})^2 = 1.979\,\mathrm{A \cdot m^2}$.

(a) Both currents are clockwise in Fig. 28–51 so, according to the right-hand rule, both dipole moments are directed into the page. The magnitude of the net dipole moment is the sum of the magnitudes of the individual moments: $\mu_{\text{net}} = \mu_i + \mu_o = 0.880\,\mathrm{A \cdot m^2} + 1.979\,\mathrm{A \cdot m^2} = 2.86\,\mathrm{A \cdot m^2}$. The net dipole moment is directed into the page.

(b) Now the dipole moment of the inner loop is directed out of the page. The moments are in opposite directions, so the magnitude of the net moment is $\mu_{\text{net}} = \mu_o - \mu_i = 1.979\,\mathrm{A \cdot m^2} - 0.880\,\mathrm{A \cdot m^2} = 1.10\,\mathrm{A \cdot m^2}$. The net dipole moment is again into the page.

63

If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi(L/2\pi N)^2 = L^2/4\pi N^2$. For maximum torque, orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular to the field. The magnitude of the torque is then

$$\tau = NiAB = (Ni)\left(\frac{L^2}{4\pi N^2}\right)B = \frac{iL^2B}{4\pi N}\,.$$

To maximize the torque, take N to have the smallest possible value, 1. Then

$$\tau = \frac{iL^2B}{4\pi} = \frac{(4.51 \times 10^{-3}\,\mathrm{A})(0.250\,\mathrm{m})^2(5.71 \times 10^{-3}\,\mathrm{T})}{4\pi} = 1.28 \times 10^{-7}\,\mathrm{N \cdot m}\,.$$

65

(a) the magnetic potential energy is given by $U = -\vec{\mu} \cdot \vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and $\vec{B}$ is the magnetic field. The magnitude of the magnetic moment is $\mu = NiA$, where i is the current in the coil, A is the area of the coil, and N is the number of turns. The moment is in the negative y direction, as you can tell by wrapping the fingers of your right hand around the coil in the direction of the current. Your thumb is then in the negative y direction. Thus $\vec{\mu} = -(3.00)(2.00\,\text{A})(4.00 \times 10^{-3}\,\text{m}^2)\,\hat{\text{j}} = -(2.40 \times 10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\text{j}}$. The magnetic potential energy is

$$\begin{aligned} U &= -(\mu_y\,\hat{\text{j}}) \cdot (B_x\,\hat{\text{i}} + B_y\,\hat{\text{j}} + B_z\,\hat{\text{k}}) = -\mu_y By \\ &= -(-2.40 \times 10^{-2}\,\text{A}\cdot\text{m}^2)(-3.00 \times 10^{-3}\,\text{T}) = -7.20 \times 10^{-5}\,\text{J}\,, \end{aligned}$$

where $\hat{\text{j}} \cdot \hat{\text{i}} = 0$, $\hat{\text{j}} \cdot \hat{\text{j}} = 1$, and $\hat{\text{j}} \cdot \hat{\text{k}} = 0$ were used.

(b) The magnetic torque on the coil is

$$\begin{aligned} \vec{\tau} &= \vec{\mu} \times \vec{B} = (\mu_y\,\hat{\text{j}}) \times (B_x\,\hat{\text{i}} + B_y\,\hat{\text{j}} + B_z\,\hat{\text{k}}) = \mu_y B_z\,\hat{\text{i}} - \mu_y B_x\,\hat{\text{k}} \\ &= (-2.40 \times 10^{-2}\,\text{A}\cdot\text{m}^2)(-4.00 \times 10^{-3}\,\text{T})\,\hat{\text{i}} - (-2.40 \times 10^{-2}\,\text{A}\cdot\text{m}^2)(2.00 \times 10^{-3}\,\text{T})\,\hat{\text{k}} \\ &= (9.6 \times 10^{-5}\,\text{N}\cdot\text{m})\,\hat{\text{i}} + (4.80 \times 10^{-5}\,\text{N}\cdot\text{m})\hat{\text{k}}\,, \end{aligned}$$

where $\hat{\text{j}} \times \hat{\text{i}} = -\hat{\text{k}}$, $\hat{\text{j}} \times \hat{\text{j}} = 0$, and $\hat{\text{j}} \times \hat{\text{k}} = \hat{\text{i}}$ were used.

73

The net force on the electron is given by $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the electron's velocity. Since the electron moves with constant velocity you know that the net force must vanish. Thus

$$\vec{E} = -\vec{v} \times \vec{B} = -(v\,\hat{\text{i}}) \times (B\,\hat{\text{k}}) = -vB\,\hat{\text{j}} = -(100\,\text{m/s})(5.00\,\text{T})\,\hat{\text{j}} = (500\,\text{V/m})\,\hat{\text{j}}\,.$$

75

(a) and (b) Suppose the particles are accelerated from rest through an electric potential difference V. Since energy is conserved the kinetic energy of a particle is $K = qV$, where q is the particle's charge. The ratio of the proton's kinetic energy to the alpha particle's kinetic energy is $K_p/K_\alpha = e/2e = 0.50$. The ratio of the deuteron's kinetic energy to the alpha particle's kinetic energy is $K_d/K_\alpha = e/2e = 0.50$.

(c) The magnitude of the magnetic force on a particle is qvB and, according to Newton's second law, this must equal mv^2/R, where v is its speed and R is the radius of its orbit. Since $v = \sqrt{2K/m} = \sqrt{2qV/m}$,

$$R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2m}{qB^2}}\,.$$

The ratio of the radius of the deuteron's path to the radius of the proton's path is

$$\frac{R_d}{R_p} = \sqrt{\frac{2.0\,\text{u}}{1.0\,\text{u}}}\sqrt{\frac{e}{e}} = 1.4\,.$$

Since the radius of the proton's path is 10 cm, the radius of the deuteron's path is (1.4)(10 cm) = 14 cm.

(d) The ratio of the radius of the alpha particle's path to the radius of the proton's path is

$$\frac{R_\alpha}{R_p} = \sqrt{\frac{4.0\,\mathrm{u}}{1.0\,\mathrm{u}}}\sqrt{\frac{e}{2e}} = 1.4\,.$$

Since the radius of the proton's path is 10 cm, the radius of the deuteron's path is (1.4)(10 cm) = 14 cm.

77

Take the velocity of the particle to be $\vec{v} = v_x\,\hat{\mathrm{i}} + v_y\,\hat{\mathrm{j}}$ and the magnetic field to be $B\,\hat{\mathrm{i}}$. The magnetic force on the particle is then

$$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\,\hat{\mathrm{i}} + v_y\,\hat{\mathrm{j}})\times(B\,\hat{\mathrm{i}}) = -qv_yB\,\hat{\mathrm{k}}\,,$$

where q is the charge of the particle. We used $\hat{\mathrm{i}}\times\hat{\mathrm{i}} = 0$ and $\hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}}$. The charge is

$$q = \frac{F}{-v_yB} = \frac{0.48\,\mathrm{N}}{-(4.0\times10^3\,\mathrm{m/s})(\sin 37^\circ)(5.0\times10^{-3}\,\mathrm{T})} = -4.0\times10^{-2}\,\mathrm{C}\,.$$

81

(a) If K is the kinetic energy of the electron and m is its mass, then its speed is

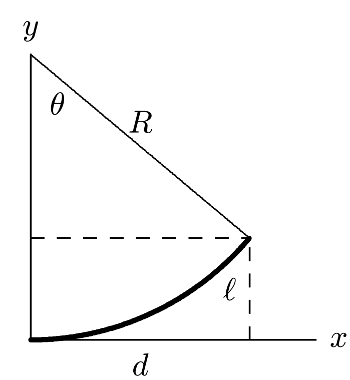
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(12\times10^3\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{9.11\times10^{-31}}} = 6.49\times10^7\,\mathrm{m/s}\,.$$

Since the electron is traveling along a line that is parallel to the horizontal component of Earth's magnetic field, that component does not enter into the calculation of the magnetic force on the electron. The magnitude of the force on the electron is evB and since $F = ma$, where a is the magnitude of its acceleration, $evB = ma$ and

$$a = \frac{evB}{m} = \frac{(1.60\times10^{-19}\,\mathrm{C})(6.49\times10^7\,\mathrm{m/s})(55.0\times10^{-6}\,\mathrm{T})}{9.11\times10^{-31}\,\mathrm{kg}} = 6.3\times10^{14}\,\mathrm{m/s^2}\,.$$

(b) If the electron does not get far from the x axis we may neglect the influence of the horizontal component of Earth's field and assume the electron follows a circular path. Its acceleration is given by $a = v^2/R$, where R is the radius of the path. Thus

$$R = \frac{v^2}{a} = \frac{(6.49\times10^7\,\mathrm{m/s})^2}{6.27\times10^{14}\,\mathrm{m/s^2}} = 6.72\,\mathrm{m}\,.$$



The solid curve on the diagram is the path. Suppose it subtends the angle θ at its center. d (= 0.200 m) is the distance traveled along the x axis and ℓ is the deflection. The right triangle yields $d = R\sin\theta$, so $\sin\theta = d/R$ and $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(x/R)^2}$. The triangle also gives $\ell = R - R\cos\theta$, so $\ell = R - R\sqrt{1-(x/R)^2}$. Substitute $R = 6.72$ m and $d = 0.2$ m to obtain $\ell = 0.0030$ m.

Chapter 29

7

(a) If the currents are parallel, the two magnetic fields are in opposite directions in the region between the wires. Since the currents are the same, the net field is zero along the line that runs halfway between the wires. There is no possible current for which the field does not vanish. If there is to be a field on the bisecting line the currents must be in opposite directions. Then the fields are in the same direction in the region between the wires.

(b) At a point halfway between the wires, the fields have the same magnitude, $\mu_0 i/2\pi r$. Thus the net field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\,\mathrm{m})(300\times10^{-6}\,\mathrm{T})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 30\,\mathrm{A}\,.$$

15

Sum the fields of the two straight wires and the circular arc. Look at the derivation of the expression for the field of a long straight wire, leading to Eq. 29–6. Since the wires we are considering are infinite in only one direction, the field of either of them is half the field of an infinite wire. That is, the magnitude is $\mu_0 i/4\pi R$, where R is the distance from the end of the wire to the center of the arc. It is the radius of the arc. The fields of both wires are out of the page at the center of the arc.

Now find an expression for the field of the arc at its center. Divide the arc into infinitesimal segments. Each segment produces a field in the same direction. If ds is the length of a segment, the magnitude of the field it produces at the arc center is $(\mu_0 i/4\pi R^2)\,ds$. If θ is the angle subtended by the arc in radians, then $R\theta$ is the length of the arc and the net field of the arc is $\mu_0 i\theta/4\pi R$. For the arc of the diagram, the field is into the page. The net field at the center, due to the wires and arc together, is

$$B = \frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i}{4\pi R} - \frac{\mu_0 i\theta}{4\pi R} = \frac{\mu_0 i}{4\pi R}(2-\theta)\,.$$

For this to vanish, θ must be exactly 2 radians.

19

Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

$$\begin{aligned} B_{\text{net}} &= 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45^\circ = \frac{2\mu_0 i}{\pi a} \\ &= \frac{2(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(20\,\text{A})}{\pi(0.20\,\text{m})} = 8.0 \times 10^{-5}\,\text{T}\,. \end{aligned}$$

In the calculation $\cos 45^\circ$ was replaced with $1/\sqrt{2}$. In unit vector notation $\vec{B} = (8.0 \times 10^{-5}\,\text{T})\,\hat{\jmath}$.

21

Follow the same steps as in the solution of Problem 17 above but change the lower limit of integration to $-L$, and the upper limit to 0. The magnitude of the net field is

$$\begin{aligned} B &= \frac{\mu_0 i R}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi}\,\frac{1}{R^2}\,\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\,\frac{L}{\sqrt{L^2+R^2}} \\ &= \frac{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.693\,\text{A})}{4\pi(0.251\,\text{m})}\,\frac{0.136\,\text{m}}{\sqrt{(0.136\,\text{m})^2+(0.251\,\text{m})^2}} = 1.32 \times 10^{-7}\,\text{T}\,. \end{aligned}$$

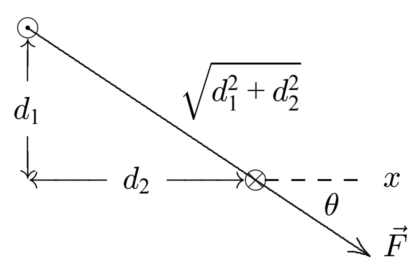
31

The current per unit width of the strip is i/w and the current through a width dx is $(i/w)\,dx$. Treat this as a long straight wire. The magnitude of the field it produces at a point that is a distance d from the edge of the strip is $dB = (\mu_0/2\pi)(i/w)\,dx/x$ and the net field is

$$\begin{aligned} B &= \frac{\mu_0 i}{2\pi w}\int_{d}^{d+w}\frac{dx}{x} = \frac{\mu_0}{2\pi w}\ln\frac{d+w}{d} \\ &= \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(4.61 \times 10^{-6}\,\text{A})}{2\pi(0.0491\,\text{m})}\ln\frac{0.0216\,\text{m}+0.0491\,\text{m}}{0.0216\,\text{m}} \\ &= 2.23 \times 10^{-11}\,\text{T}\,. \end{aligned}$$

35

The magnitude of the force of wire 1 on wire 2 is given by $\mu_0 i_1 i_2/2\pi r$, where i_1 is the current in wire 1, i_2 is the current in wire 2, and r is the separation of the wires. The distance between the wires is $r = \sqrt{d_1^2 + d_2^2}$. Since the currents are in opposite directions the wires repel each other so the force on wire 2 is along the line that joins the wires and is away from wire 1.



To find the x component of the force, multiply the magnitude of the force by the cosine of the angle θ that the force makes with the x axis. This is $\cos\theta = d_2/\sqrt{d_1^2 + d_2^2}$. Thus the x component of the force is

$$F_x = \frac{\mu_0 i_1 i_2}{2\pi}\frac{d_2}{d_1^2 + d_2^2}$$
$$= \frac{(2\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(4.00 \times 10^{-3}\,\mathrm{A})(6.80 \times 10^{-3}\,\mathrm{A})}{2\pi}\frac{0.0500\,\mathrm{m}}{(0.024\,\mathrm{m})^2 + (5.00\,\mathrm{m})^2}$$
$$= 8.84 \times 10^{-11}\,\mathrm{T}\,.$$

43

(a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus $i_{\text{enc}} = -i$ and

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(2.0\,\mathrm{A}) = -2.5 \times 10^{-6}\,\mathrm{T\cdot m}\,.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

53

(a) Assume that the point is inside the solenoid. The field of the solenoid at the point is parallel to the solenoid axis and the field of the wire is perpendicular to the solenoid axis. The net field makes an angle of 45° with the axis if these two fields have equal magnitudes.

The magnitude of the magnetic field produced by a solenoid at a point inside is given by $B_{\text{sol}} = \mu_0 i_{\text{sol}} n$, where n is the number of turns per unit length and i_{sol} is the current in the solenoid. The magnitude of the magnetic field produced by a long straight wire at a point a distance r away is given by $B_{\text{wire}} = \mu_0 i_{\text{wire}}/2\pi r$, where i_{wire} is the current in the wire. We want $\mu_0 n i_{\text{sol}} = \mu_0 i_{\text{wire}}/2\pi r$. The solution for r is

$$r = \frac{i_{\text{wire}}}{2\pi n i_{\text{sol}}} = \frac{6.00\,\mathrm{A}}{2\pi(10.0 \times 10^2\,\mathrm{m^{-1}})(20.0 \times 10^{-3}\,\mathrm{A})} = 4.77 \times 10^{-2}\,\mathrm{m} = 4.77\,\mathrm{cm}\,.$$

This distance is less than the radius of the solenoid, so the point is indeed inside as we assumed.

(b) The magnitude of the either field at the point is

$$B_{\text{sol}} = B_{\text{wire}} = \mu_0 n i_{\text{sol}} = (4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(10.0 \times 10^2\,\mathrm{m^{-1}})(20.0 \times 10^{-3}\,\mathrm{A}) = 2.51 \times 10^{-5}\,\mathrm{T}\,.$$

Each of the two fields is a vector component of the net field, so the magnitude of the net field is the square root of the sum of the squares of the individual fields: $B = \sqrt{2(2.51 \times 10^{-5}\,\mathrm{T})^2} = 3.55 \times 10^{-5}\,\mathrm{T}$.

57

The magnitude of the dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. Use $A = \pi R^2$, where R is the radius. Thus

$$\mu = Ni\pi R^2 = (200)(0.30\,\mathrm{A})\pi(0.050\,\mathrm{m})^2 = 0.47\,\mathrm{A\cdot m^2}\,.$$

65

(a) Take the magnetic field at a point within the hole to be the sum of the fields due to two current distributions. The first is the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder with the hole. The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but it is in the opposite direction. Notice that if these two situations are superposed, the total current in the region of the hole is zero.

Recall that a solid cylinder carrying current i, uniformly distributed over a cross section, produces a magnetic field with magnitude $B = \mu_0 i r / 2\pi R^2$ a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder.

For the cylinder of this problem, the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)} ,$$

where A $(= \pi(a^2 - b^2))$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$i_1 = JA_1 = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 i_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2(a^2 - b^2)} = \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} .$$

The current in the cylinder that fills the hole is

$$i_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 i_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2(a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} .$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(5.25\,\mathrm{A})(0.0200\,\mathrm{m})}{2\pi[(0.0400\,\mathrm{m})^2 - (0.0150\,\mathrm{m})^2]} = 1.53 \times 10^{-5}\,\mathrm{T}$$

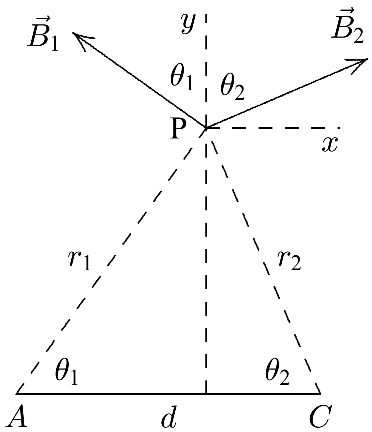
for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$, the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2} .$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$, the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

(c) The diagram shows the situation in a cross-sectional plane of the cylinder. P is a point within the hole, A is on the axis of the cylinder, and C is on the axis of the hole. The magnetic field due to the cylinder without the hole, carrying a uniform current out of the page, is labeled $\vec{B}_1$ and the magnetic field of the cylinder that fills the hole, carrying a uniform current into the page, is labeled $\vec{B}_2$. The line from A to P makes the angle θ_1 with the line that joins the centers of the cylinders and the line from C to P makes the angle θ_2 with that line, as shown. $\vec{B}_1$ is perpendicular to the line from A to P and so makes the angle θ_1 with the vertical. Similarly, $\vec{B}_2$ is perpendicular to the line from C to P and so makes the angle θ_2 with the vertical.



The x component of the total field is

$$B_x = B_2 \sin\theta_2 - B_1 \sin\theta_1 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \sin\theta_2 - \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \sin\theta_1$$
$$= \frac{\mu_0 i}{2\pi(a^2 - b^2)} [r_2 \sin\theta_2 - r_1 \sin\theta_1] \,.$$

As the diagram shows, $r_2 \sin\theta_2 = r_1 \sin\theta_1$, so $B_x = 0$. The y component is given by

$$B_y = B_2 \cos\theta_2 + B_1 \cos\theta_1 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \cos\theta_2 + \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \cos\theta_1$$
$$= \frac{\mu_0 i}{2\pi(a^2 - b^2)} [r_2 \cos\theta_2 + r_1 \cos\theta_1] \,.$$

The diagram shows that $r_2 \cos\theta_2 + r_1 \cos\theta_1 = d$, so

$$B_y = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} \,.$$

This is identical to the result found in part (a) for the field on the axis of the hole. It is independent of r_1, r_2, θ_1, and θ_2, showing that the field is uniform in the hole.

<u>**71**</u>

Use the Biot-Savart law in the form

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\Delta\vec{s} \times \vec{r}}{r^3} \,.$$

Take $\Delta\vec{s}$ to be $\Delta s\,\hat{j}$, and $\vec{r}$ to be $x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$. Then $\Delta\vec{s} \times \vec{r} = \Delta s\,\hat{j} \times (x\,\hat{i} + y\,\hat{j} + z\,\hat{k}) = \Delta s(z\,\hat{i} - x\,\hat{k})$, where $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{j} \times \hat{j} = 0$, and $\hat{j} \times \hat{k} = \hat{i}$ were used. In addition, $r = \sqrt{x^2 + y^2 + z^2}$. The Biot-Savart equation becomes

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\Delta s(z\,\hat{i} - z\,\hat{k})}{(x^2 + y^2 + z^2)^{3/2}} \,.$$

(a) For $x = 0$, $y = 0$, and $z = 5.0\,\text{m}$,

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}}{4\pi}\,\frac{(2.0\,\text{A})(0.030\,\text{m})(5.0\,\text{m})\,\hat{\imath}}{(5.0\,\text{m})^3} = (2.4 \times 10^{-10}\,\text{T})\,\hat{\imath}\,.$$

(b) For $x = 0$, $y = 6.0\,\text{m}$, and $z = 0$, $\vec{B} = 0$.

(c) For $x = 7.0\,\text{m}$, $y = 7.0\,\text{m}$, and $z = 0$,

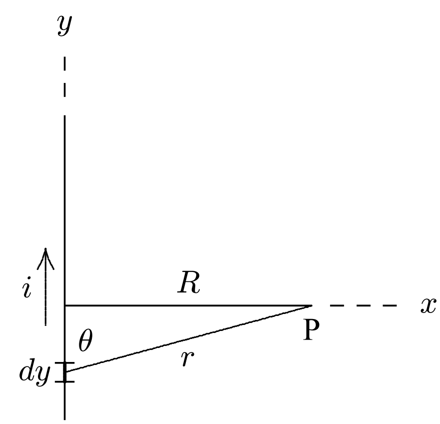
$$\vec{B} = \frac{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}}{4\pi}\,\frac{(2.0\,\text{A})(0.030\,\text{m})(-7.0\,\text{m})\,\hat{k}}{[(7.0\,\text{m})^2 + (7.0\,\text{m})^2]^{3/2}} = (4.3 \times 10^{-11}\,\text{T})\,\hat{k}\,.$$

(d) For $x = -3.0\,\text{m}$, $y = -4.0\,\text{m}$, and $z = 0$,

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}}{4\pi}\,\frac{(2.0\,\text{A})(0.030\,\text{m})(3.0\,\text{m})\,\hat{k}}{[(-3.0\,\text{m})^2 + (-4.0\,\text{m})^2]^{3/2}} = (1.4 \times 10^{-10}\,\text{T})\,\hat{k}\,.$$

<u>77</u>

First consider the finite wire segment shown on the right. It extends from $y = -d$ to $y = a-d$, where a is the length of the segment, and it carries current i in the positive y direction. Let dy be an infinitesimal length of wire at coordinate y. According to the Biot-Savart law the magnitude of the magnetic field at P due to this infinitesimal length is $dB = (\mu_0/4\pi)(i\sin\theta/r^2)\,dy$. Now $r^2 = y^2 + R^2$ and $\sin\theta = R/r = R/\sqrt{y^2+R^2}$, so



$$dB = \frac{\mu_0}{4\pi}\,\frac{iR}{(y^2+R^2)^{3/2}}dy$$

and the field of the entire segment is

$$B = \frac{\mu_0}{4\pi}\,iR\int_{-d}^{a-d}\frac{y}{(y^2+R^2)^{3/2}}\,dy = \mu_0/4\pi\,\frac{i}{R}\left[\frac{a-d}{\sqrt{R^2+(a-d)^2}} + \frac{d}{\sqrt{R^2+d^2}}\right],$$

where integral 19 of Appendix E was used.

All four sides of the square produce magnetic fields that are into the page at P, so we sum their magnitudes. To calculate the field of the left side of the square put $d = 3a/4$ and $R = a/4$. The result is

$$B_{\text{left}} = \frac{\mu_0}{4\pi}\,\frac{4i}{a}\left[\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{10}}\right] = \frac{\mu_O}{3\pi}\,\frac{4i}{a}(1.66)\,.$$

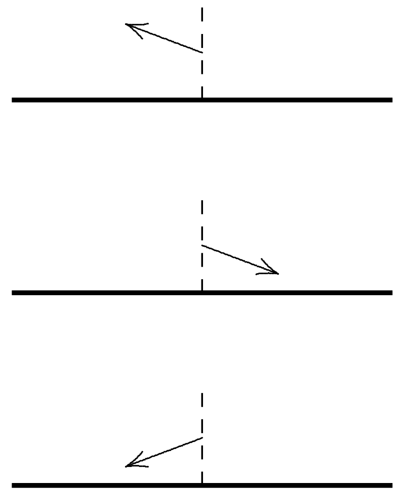
The field of the upper side of the square is the same. To calculate the field of the right side of the square put $d = a/4$ and $R = 3a/4$. The result is

$$B_{\text{right}} = \frac{\mu_0}{4\pi}\,\frac{4i}{3a}\left[\frac{3}{\sqrt{18}} + \frac{1}{\sqrt{10}}\right] = \frac{\mu_O}{3\pi}\,\frac{4i}{a}(0.341)\,.$$

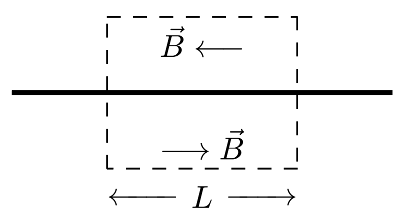
The field of the bottom side is the same. The total field at P is

$$B = B_{\text{left}} + B_{\text{upper}} + B_{\text{right}} + B_{\text{lower}} = \frac{\mu_0}{4\pi}\frac{4i}{a}(1.66 + 1.66 + 0.341 + 0.341)$$
$$= \frac{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}}{4\pi}\frac{4(10\,\text{A})}{0.080\,\text{m}}(4.00) = 2.0 \times 10^{-4}\,\text{T}\,.$$

79



(a) Suppose the field is not parallel to the sheet, as shown in the upper diagram. Reverse the direction of the current. According to the Biot-Savart law, the field reverses, so it will be as in the second diagram. Now rotate the sheet by 180° about a line that is perpendicular to the sheet. The field, of course, will rotate with it and end up in the direction shown in the third diagram. The current distribution is now exactly as it was originally, so the field must also be as it was originally. But it is not. Only if the field is parallel to the sheet will be final direction of the field be the same as the original direction. If the current is out of the page, any infinitesimal portion of the sheet in the form of a long straight wire produces a field that is to the left above the sheet and to the right below the sheet. The field must be as drawn in Fig. 29–85.



(b) Integrate the tangential component of the magnetic field around the rectangular loop shown with dotted lines. The upper and lower edges are the same distance from the current sheet and each has length L. This means the field has the same magnitude along these edges. It points to the left along the upper edge and to the right along the lower.

If the integration is carried out in the counterclockwise sense, the contribution of the upper edge is BL, the contribution of the lower edge is also BL, and the contribution of each of the sides is zero because the field is perpendicular to the sides. Thus $\oint \vec{B}\cdot d\vec{s} = 2BL$. The total current through the loop is λL. Ampere's law yields $2BL = \mu_0\lambda L$, so $B = \mu_0\lambda/2$.

81

(a) Use a circular Amperian path that has radius r and is concentric with the cylindrical shell as shown by the dotted circle on Fig. 29–86. The magnetic field is tangent to the path and has uniform magnitude on it, so the integral on the left side of the Ampere's law equation is $\oint \vec{B}\cdot d\vec{s} = 2\pi r B$. The current through the Amperian path is the current through the region outside the circle of radius b and inside the circle of radius r. Since the current is uniformly distributed through a cross section of the shell, the enclosed current is $i(r^2 - b^2)/(a^2 - b^2)$. Thus

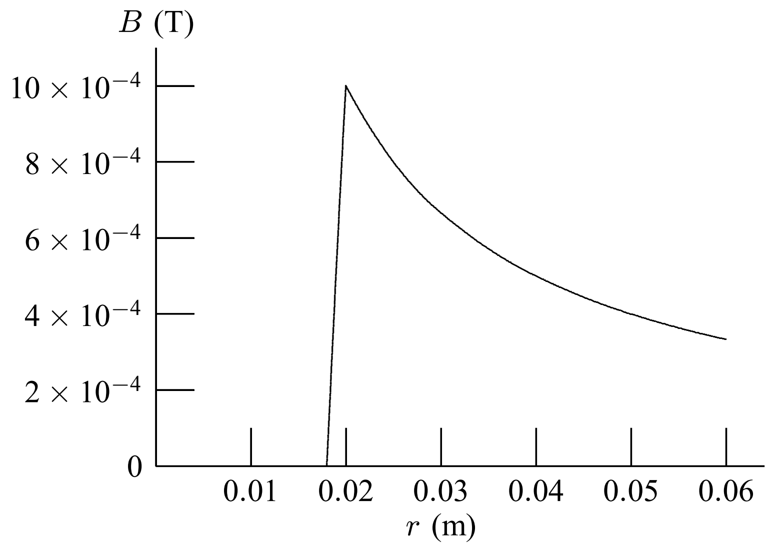
$$2\pi r B = \frac{r^2 - b^2}{a^2 - b^2}\, i$$

and

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \frac{r^2 - b^2}{r}.$$

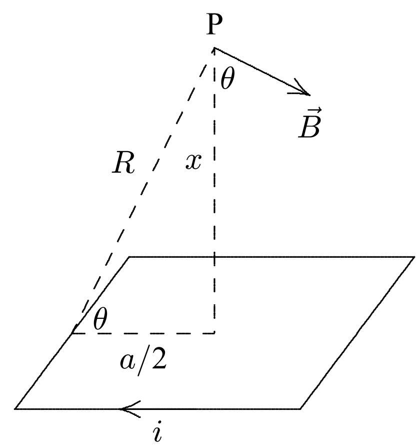
(b) When $r = a$ this expression reduce to $B = \mu_0 i/2\pi r$, which is the correct expression for the field of a long straight wire. When $r = b$ it reduces to $B = 0$, which is correct since there is no field inside the shell. When $b = 0$ it reduces to $B = \mu_0 ir/2\pi a^2$, which is correct for the field inside a cylindrical conductor.

(c) The graph is shown below.



89

The result of Problem 11 is used four times, once for each of the sides of the square loop. A point on the axis of the loop is also on a perpendicular bisector of each of the loop sides. The diagram shows the field due to one of the loop sides, the one on the left. In the expression found in Problem 11, replace L with a and R with $\sqrt{x^2 + a^2/4} = \frac{1}{2}\sqrt{4x^2 + a^2}$. The field due to the side is therefore



$$B = \frac{\mu_0 ia}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}.$$

The field is in the plane of the dotted triangle shown and is perpendicular to the line from the midpoint of the loop side to the point P. Therefore it makes the angle θ with the vertical.

When the fields of the four sides are summed vectorially the horizontal components add to zero. The vertical components are all the same, so the total field is given by

$$B_{\text{total}} = 4B\cos\theta = \frac{4Ba}{2R} = \frac{4Ba}{\sqrt{4x^2 + a^2}}.$$

Thus

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}} .$$

For $x = 0$, the expression reduces to

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi a^2 \sqrt{2} a} = \frac{2\sqrt{2}\mu_0 i}{\pi a} ,$$

in agreement with the result of Problem 12.

91

Use Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where the integral is around a closed loop and i_{enc} is the net current through the loop. For the dashed loop shown on the diagram $i = 0$. Assume the integral $\int \vec{B} \cdot d\vec{s}$ is zero along the bottom, right, and top sides of the loop as it would be if the field lines are as shown on the diagram. Along the right side the field is zero and along the top and bottom sides the field is perpendicular to $d\vec{s}$. If ℓ is the length of the left edge, then direct integration yields $\oint \vec{B} \cdot d\vec{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not precipitously as shown.

Chapter 30

5

The magnitude of the magnetic field inside the solenoid is $B = \mu_0 n i_s$, where n is the number of turns per unit length and i_s is the current. The field is parallel to the solenoid axis, so the flux through a cross section of the solenoid is $\Phi_B = A_s B = \mu_0 \pi r_s^2 n i_s$, where A_s $(= \pi r_s^2)$ is the cross-sectional area of the solenoid. Since the magnetic field is zero outside the solenoid, this is also the flux through the coil. The emf in the coil has magnitude

$$\mathcal{E} = \frac{N d\Phi_B}{dt} = \mu_0 \pi r_s^2 N n \frac{di_s}{dt}$$

and the current in the coil is

$$i_c = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi r_s^2 N n}{R} \frac{di_s}{dt},$$

where N is the number of turns in the coil and R is the resistance of the coil. The current changes linearly by 3.0 A in 50 ms, so $di_s/dt = (3.0\,\text{A})/(50 \times 10^{-3}\,\text{s}) = 60\,\text{A/s}$. Thus

$$i_c = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\pi(0.016\,\text{m})^2(120)(220 \times 10^2\,\text{m}^{-1})}{5.3\,\Omega}(60\,\text{A/s}) = 3.0 \times 10^{-2}\,\text{A}.$$

21

(a) In the region of the smaller loop, the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 29–26, with $z = x$ and much greater than R, gives

$$B = \frac{\mu_0 i R^2}{2x^3}$$

for the magnitude. The field is upward in the diagram. The magnetic flux through the smaller loop is the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi \mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) The field of the larger loop is upward and decreases with distance away from the loop. As the smaller loop moves away, the flux through it decreases. The induced current is directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It is counterclockwise as viewed from above, in the same direction as the current in the larger loop.

29

Thermal energy is generated at the rate $\mathcal{E}^2/R$, where $\mathcal{E}$ is the emf in the wire and R is the resistance of the wire. The resistance is given by $R = \rho L/A$, where ρ is the resistivity of copper, L is the length of the wire, and A is the cross-sectional area of the wire. The resistivity can be found in Table 26–1. Thus

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(0.500\,\text{m})}{\pi(0.500 \times 10^{-3}\,\text{m})^2} = 1.076 \times 10^{-2}\,\Omega\,.$$

Faraday's law is used to find the emf. If B is the magnitude of the magnetic field through the loop, then $\mathcal{E} = A\,dB/dt$, where A is the area of the loop. The radius r of the loop is $r = L/2\pi$ and its area is $\pi r^2 = \pi L^2/4\pi^2 = L^2/4\pi$. Thus

$$\mathcal{E} = \frac{L^2}{4\pi}\frac{dB}{dt} = \frac{(0.500\,\text{m})^2}{4\pi}(10.0 \times 10^{-3}\,\text{T/s}) = 1.989 \times 10^{-4}\,\text{V}\,.$$

The rate of thermal energy generation is

$$P = \frac{\mathcal{E}^2}{R} = \frac{(1.989 \times 10^{-4}\,\text{V})^2}{1.076 \times 10^{-2}\,\Omega} = 3.68 \times 10^{-6}\,\text{W}\,.$$

37

(a) The field point is inside the solenoid, so Eq. 30–25 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5 \times 10^{-3}\,\text{T/s})(0.0220\,\text{m}) = 7.15 \times 10^{-5}\,\text{V/m}\,.$$

(b) Now the field point is outside the solenoid and Eq. 30–27 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5 \times 10^{-3}\,\text{T/s})\frac{(0.0600\,\text{m})^2}{(0.0820\,\text{m})} = 1.43 \times 10^{-4}\,\text{V/m}\,.$$

51

Starting with zero current when the switch is closed, at time $t = 0$, the current in an RL series circuit at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

where τ_L is the inductive time constant, $\mathcal{E}$ is the emf, and R is the resistance. You want to calculate the time t for which $i = 0.9990\mathcal{E}/R$. This means

$$0.9990\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

so

$$0.9990 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.0010\,.$$

Take the natural logarithm of both sides to obtain $-(t/\tau_L) = \ln(0.0010) = -6.91$. That is, 6.91 inductive time constants must elapse.

55

(a) If the battery is switched into the circuit at time $t = 0$, then the current at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

where $\tau_L = L/R$. You want to find the time for which $i = 0.800\mathcal{E}/R$. This means

$$0.800 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.200\,.$$

Take the natural logarithm of both sides to obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\mathrm{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\mathrm{s}\,.$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\mathrm{V}}{1.20\times10^{3}\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37\times10^{-3}\,\mathrm{A}\,.$$

59

(a) Assume i is from left to right through the closed switch. Let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor and also take it to be downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$. Since $di/dt = 0$, the junction rule yields $(di_1/dt) = -(di_2/dt)$. Substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1R = 0\,.$$

This equation is similar to Eq. 30–44, and its solution is the function given as Eq. 30–45:

$$i_1 = i_0e^{-Rt/L}\,,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now, just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that time, $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L}$$

and

$$i_2 = i - i_1 = i\left[1 - e^{-Rt/L}\right] .$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} ,$$

so

$$e^{-Rt/L} = \frac{1}{2} .$$

Take the natural logarithm of both sides and use $\ln(1/2) = -\ln 2$ to obtain $(Rt/L) = \ln 2$ or

$$t = \frac{L}{R} \ln 2 .$$

63

(a) If the battery is applied at time $t = 0$, the current is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) ,$$

where $\mathcal{E}$ is the emf of the battery, R is the resistance, and τ_L is the inductive time constant. In terms of R and the inductance L, $\tau_L = L/R$. Solve the current equation for the time constant. First obtain

$$e^{-t/\tau_L} = 1 - \frac{iR}{\mathcal{E}} ,$$

then take the natural logarithm of both sides to obtain

$$-\frac{t}{\tau_L} = \ln\left[1 - \frac{iR}{\mathcal{E}}\right] .$$

Since

$$\ln\left[1 - \frac{iR}{\mathcal{E}}\right] = \ln\left[1 - \frac{(2.00 \times 10^{-3}\,\mathrm{A})(10.0 \times 10^{3}\,\Omega)}{50.0\,\mathrm{V}}\right] = -0.5108 ,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00 \times 10^{-3}\,\mathrm{s})/(0.5108) = 9.79 \times 10^{-3}\,\mathrm{s}$ and the inductance is

$$L = \tau_L R = (9.79 \times 10^{-3}\,\mathrm{s})(10.0 \times 10^{3}\,\Omega) = 97.9\,\mathrm{H} .$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\mathrm{H})(2.00 \times 10^{-3}\,\mathrm{A})^2 = 1.96 \times 10^{-4}\,\mathrm{J} .$$

69

(a) At any point, the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid, $B = \mu_0 n i$, where n is the number of turns per unit length and i is the current. For the solenoid of this problem, $n = (950)/(0.850\,\text{m}) = 1.118 \times 10^3\,\text{m}^{-1}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1.118 \times 10^3\,\text{m}^{-1})^2(6.60\,\text{A})^2 = 34.2\,\text{J/m}^3\,.$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B V$, where V is the volume of the solenoid. V is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (34.2\,\text{J/m}^3)(17.0 \times 10^{-4}\,\text{m}^2)(0.850\,\text{m}) = 4.94 \times 10^{-2}\,\text{J}\,.$$

73

(a) The mutual inductance M is given by

$$\mathcal{E}_1 = M\,\frac{di_2}{dt}\,,$$

where $\mathcal{E}_1$ is the emf in coil 1 due to the changing current i_2 in coil 2. Thus

$$M = \frac{\mathcal{E}_1}{di_2/dt} = \frac{25.0 \times 10^{-3}\,\text{V}}{15.0\,\text{A/s}} = 1.67 \times 10^{-3}\,\text{H}\,.$$

(b) The flux linkage in coil 2 is

$$N_2\Phi_{21} = Mi_1 = (1.67 \times 10^{-3}\,\text{H})(3.60\,\text{A}) = 6.01 \times 10^{-3}\,\text{Wb}\,.$$

75

(a) Assume the current is changing at the rate di/dt and calculate the total emf across both coils. First consider the left-hand coil. The magnetic field due to the current in that coil points to the left. So does the magnetic field due to the current in coil 2. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus the emf in coil 1 is

$$\mathcal{E}_1 = -\,(L_1 + M)\,\frac{di}{dt}\,.$$

The magnetic field in coil 2 due to the current in that coil points to the left, as does the field in coil 2 due to the current in coil 1. The two sources of emf are again in the same direction and the emf in coil 2 is

$$\mathcal{E}_2 = -\,(L_2 + M)\,\frac{di}{dt}\,.$$

The total emf across both coils is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -\,(L_1 + L_2 + 2M)\,\frac{di}{dt}\,.$$

This is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 + 2M$.

(b) Reverse the leads of coil 2 so the current enters at the back of the coil rather than the front as pictured in the diagram. Then the field produced by coil 2 at the site of coil 1 is opposite the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\mathcal{E}_1 = -(L_1 - M)\frac{di}{dt}.$$

Similarly the emf across coil 2 is

$$\mathcal{E}_2 = -(L_2 - M)\frac{di}{dt}.$$

The total emf across both coils is

$$\mathcal{E} = -(L_1 + L_2 - 2M)\frac{di}{dt}.$$

This the same as the emf that would be produced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 - 2M$.

79

(a) The electric field lines are circles that are concentric with the cylindrical region and the magnitude of the field is uniform around any circle. Thus the emf around a circle of radius r is $\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = 2\pi r E$. Here r is inside the cylindrical region so the magnetic flux is $\pi r^2 B$. According to Faraday's law $2\pi r E = -\pi r^2 (dB/dt)$ and

$$E = -\tfrac{1}{2} r \frac{dB}{dt} = -\tfrac{1}{2}(0.050\,\text{m})(-10 \times 10^{-3}\,\text{T/s}) = 2.5 \times 10^{-4}\,\text{V/m}.$$

Since the normal used to compute the flux was taken to be into the page, in the direction of the magnetic field, the positive direction for the electric is clockwise. The calculated value of E is positive, so the electric field at point a is toward the left and $\vec{E} = -(2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}$.

The force on the electron is $\vec{F} = -e\vec{E}$ and, according to Newton's second law, its acceleration is

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m} = -\frac{(1.60 \times 10^{-19}\,\text{C})(-2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}}{9.11 \times 10^{-31}\,\text{kg}} = (4.4 \times 10^{7}\,\text{m/s}^2)\,\hat{\imath}.$$

The mass and charge of an electron can be found in Appendix B.

(b) The electric field at $r = 0$ is zero, so the force and acceleration of an election placed at point b are zero.

(c) The electric field at point c has the same magnitude as the field at point a but now the field is to the right. That is $\vec{E} = (2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}$ and $\vec{a} = -(4.4 \times 10^{7}\,\text{m/s}^2)\,\hat{\imath}$.

81

(a) The magnetic flux through the loop is $\Phi_B = BA$, where B is the magnitude of the magnetic field and A is the area of the loop. The magnitude of the average emf is given by Faraday's law : $\mathcal{E}_{\text{avg}} = B\Delta A/\Delta t$, where ΔA is the change in the area in time Δt. Since the final area is zero, the change in area is the initial area and $\mathcal{E}_{\text{avg}} = BA/\Delta t = (2.0\,\text{T})(0.20\,\text{m})^2/(0.20\,\text{s}) = 0.40\,\text{V}$.

(b) The average current in the loop is the emf divided by the resistance of the loop: $i_{avg} = \mathcal{E}_{avg}/R = (0.40\,\text{V})/(20 \times 10^{-3}\,\Omega) = 20\,\text{A}$.

85

(a), (b), (c), (d), and (e) Just after the switch is closed the current i_2 through the inductor is zero. The loop rule applied to the left loop gives $\mathcal{E} - I_1R_1 = 0$, so $i_1 = \mathcal{E}/R_1 = (10\,\text{V})/(5.0\,\Omega) = 2.0\,\text{A}$. The junction rule gives $i_s = i_1 = 2.0\,\text{A}$. Since $i_2 = 0$, the potential difference across R_2 is $V_2 = i_2R_2 = 0$. The potential differences across the inductor and resistor must sum to $\mathcal{E}$ and, since $V_2 = 0$, $V_L = \mathcal{E} = 10\,\text{V}$. The rate of change of i_2 is $di_2/dt = V_L/L = (10\,\text{V})/(5.0\,\text{H}) = 2.0\,\text{A/s}$.

(g), (h), (i), (j), (k), and (l) After the switch has been closed for a long time the current i_2 reaches a constant value. Since its derivative is zero the potential difference across the inductor is $V_L = 0$. The potential differences across both R_1 and R_2 are equal to the emf of the battery, so $i_1 = \mathcal{E}/R_1 = (10\,\text{V})/(5.0\,\Omega) = 2.0\,\text{A}$ and $i_2 = \mathcal{E}/R_2 = (10\,\text{V})/(10\,\Omega) = 1.0\,\text{A}$. The junction rule gives $i_s = i_1 + i_2 = 3.0\,\text{A}$.

95

(a) Because the inductor is in series with the battery the current in the circuit builds slowly and just after the switch is closed it is zero.

(b) Since all currents are zero just after the switch is closed the emf of the inductor must match the emf of the battery in magnitude. Thus $L(di_{bat}/dt) = \mathcal{E}$ and $di_{bat} = \mathcal{E}/L = (40\,\text{V})/(50 \times 10^{-3}\,\text{H}) = 8.0 \times 10^2\,\text{A/s}$.

(c) Replace the two resistors in parallel with their equivalent resistor. The equivalent resistance is

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{(20\,\text{k}\Omega)(20\,\text{k}\Omega)}{20\,\text{k}\Omega + 20\,\text{k}\Omega} = 10\,\text{k}\Omega\,.$$

The current as a function of time is given by

$$i_{bat} = \frac{\mathcal{E}}{R_{eq}}\left[1 - e^{-t/\tau_L}\right]\,,$$

where τ_L is the inductive time constant. Its value is $\tau_L = L/R_{eq} = (50 \times 10^{-3}\,\text{H})/(10 \times 10^3\,\Omega) = 5.0 \times 10^{-6}\,\text{s}$. At $t = 3.0 \times 10^{-6}\,\text{s}$, $t/\tau_L = (3.0)/(5.0) = 0.60$ and

$$i_{bat} = \frac{40\,\text{V}}{10 \times 10^3\,\Omega}\left[1 - e^{-0.60}\right] = 1.8 \times 10^{-3}\,\text{A}\,.$$

(d) Differentiate the expression for i_{bat} to obtain

$$\frac{di_{bat}}{dt} = \frac{\mathcal{E}}{R_{eq}}\frac{1}{\tau_L}e^{-t/\tau_L} = \frac{\mathcal{E}}{L}e^{-t/\tau_L}\,,$$

where $\tau_L = L/R_{eq}$ was used to obtain the last form. At $t = 3.0 \times 10^{-6}\,\text{s}$

$$\frac{di_{bat}}{dt} = \frac{40\,\text{V}}{50 \times 10^{-3}\,\text{H}}e^{-0.60} = 4.4 \times 10^2\,\text{A/s}\,.$$

(e) A long time after the switch is closed the currents are constant and the emf of the inductor is zero. The current in the battery is $i_{bat} = \mathcal{E}/R_{eq} = (40\,\text{V})/(10 \times 10^3\,\Omega) = 4.0 \times 10^{-3}\,\text{A}$.

(f) The currents are constant and $di_{\text{bat}}/dt = 0$.

97

(a) and (b) Take clockwise current to be positive and counterclockwise current to be negative. Then according to the right-hand rule we must take the normal to the loop to be into the page, so the flux is negative if the magnetic field is out of the page and positive if it is into the page. Assume the field in region 1 is out of the page. We will obtain a negative result for the field if the assumption is incorrect. Let x be the distance that the front edge of the loop is into region 1. Then while the loop is entering this region flux is $-B_1Hx$ and, according to Faraday's law, the emf induced around the loop is $\mathcal{E} = B_1H(dx/dt) = B_1Hv$. The current in the loop is $i = \mathcal{E}/R = B_1Hv/R$, so

$$B_1 = \frac{iR}{Hv} = \frac{(3.0 \times 10^{-6}\,\text{A})(0.020\,\Omega)}{(0.0150\,\text{m})(0.40\,\text{m/s})} = 1.0 \times 10^{-5}\,\text{T}\,.$$

The field is positive and therefore out of the page.

(c) and (d) Assume that the field B_2 of region 2 is out of the page. Let x now be the distance the front end of the loop is into region 2 as the loop enters that region. The flux is $-B_1H(D-x) - B_2Hx$, the emf is $\mathcal{E} = -B_1Hv + B_2Hv = (B_2 - B_1)Hv$, and the current is $i = (B_2 - B_1)Hv/R$. The field of region 2 is

$$B_2 = B_1 + \frac{iR}{Hv} = 1.0 \times 10^{-5}\,\text{T} + \frac{(-2.0 \times 10^{-6}\,\text{A}(0.020\,\Omega)}{(0.015\,\text{m})(0.40\,\text{m/s})} = 3.3 \times 10^{-6}\,\text{T}\,.$$

The field is positive, indicating that it is out of the page.

Chapter 31

7

(a) The mass m corresponds to the inductance, so $m = 1.25\,\mathrm{kg}$.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\,\mathrm{C}\right)^2}{2(5.70 \times 10^{-6}\,\mathrm{J})} = 2.69 \times 10^{-3}\,\mathrm{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\,\mathrm{m/N}} = 372\,\mathrm{N/m}\,.$$

(c) The maximum displacement x_m corresponds to the maximum charge, so

$$x_m = 1.75 \times 10^{-4}\,\mathrm{m}\,.$$

(d) The maximum speed v_m corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\,\mathrm{C}}{\sqrt{(1.25\,\mathrm{H})(2.69 \times 10^{-3}\,\mathrm{F})}} = 3.02 \times 10^{-3}\,\mathrm{A}\,.$$

Thus $v_m = 3.02 \times 10^{-3}\,\mathrm{m/s}$.

15

(a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Hence, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\mathrm{pF}}}{\sqrt{10\,\mathrm{pF}}} = 6.0\,.$$

(b) You want to choose the additional capacitance C so the ratio of the frequencies is

$$r = \frac{1.60\,\mathrm{MHz}}{0.54\,\mathrm{MHz}} = 2.96\,.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\mathrm{pF}}}{\sqrt{C + 10\,\mathrm{pF}}} = 2.96\,.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}\,.$$

(c) Solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency, $C = 365\,\text{pF} + 36\,\text{pF} = 401\,\text{pF}$ and $f = 0.54\,\text{MHz}$. Thus

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2(401\times10^{-12}\,\text{F})(0.54\times10^6\,\text{Hz})^2} = 2.2\times10^{-4}\,\text{H}\,.$$

27

Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\text{max }1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q^2_{\text{max }1}}{2C} = \frac{Q^2}{2C}\,e^{-Rt/L}\,,$$

where

$$q_{\text{max }1} = Q\,e^{-Rt/2L}$$

was used. Here Q is the charge at $t = 0$. One cycle later, the maximum charge is

$$q_{\text{max }2} = Q\,e^{-R(t+T)/2L}$$

and the energy is

$$U(t+T) = \frac{q^2_{\text{max }2}}{2C} = \frac{Q^2}{2C}\,e^{-R(t+T)/L}\,,$$

where T is the period of oscillation. The fractional loss in energy is

$$\frac{\Delta U}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L}\,.$$

Assume that RT/L is small compared to 1 (the resistance is small) and use the Maclaurin series to expand the exponential. The first two terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L}\,.$$

Replace T with $2\pi/\omega$, where ω is the angular frequency of oscillation. Thus

$$\frac{\Delta U}{U} \approx 1 - \left(1 - \frac{RT}{L}\right) = \frac{RT}{L} = \frac{2\pi R}{\omega L}\,.$$

33

(a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi$, where n is an integer, including zero. The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ or

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\,\text{s}^{-1})} = 6.73\times10^{-3}\,\text{s}\,.$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or $\omega_d t - 3\pi/4 = \pi/2 \pm 2n\pi$. The first time this occurs after $t = 0$ is when

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\,\text{s}^{-1})} = 1.12\times10^{-2}\,\text{s}\,.$$

(c) The current lags the inductor by $\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \mathcal{E}_m$. Thus $\mathcal{E}_m = I\omega_d L$ and

$$L = \frac{\mathcal{E}_m}{I\omega_d} = \frac{30.0\,\mathrm{V}}{(620 \times 10^{-3}\,\mathrm{A})(350\,\mathrm{rad/s})} = 0.138\,\mathrm{H}\,.$$

39

(a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\,\mathrm{Hz})(70.0 \times 10^{-6}\,\mathrm{F})} = 37.9\,\Omega\,.$$

The inductive reactance is

$$X_L = \omega_d L = 1\pi f_d L = 2\pi(60.0\,\mathrm{Hz})(230 \times 10^{-3}\,\mathrm{H}) = 86.7\,\Omega\,.$$

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 206\,\Omega\,.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{200\,\Omega}\right) = 13.7^\circ\,.$$

(c) The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\mathrm{V}}{206\,\Omega} = 0.175\,\mathrm{A}\,.$$

(d) The voltage amplitudes are

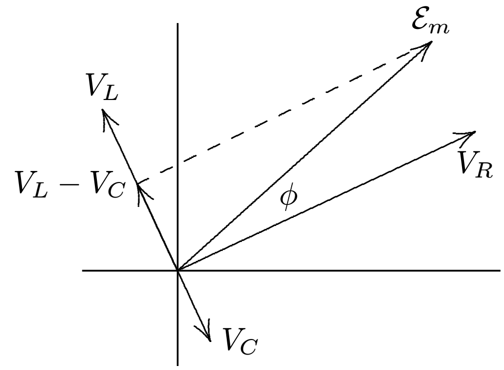
$$V_R = IR = (0.175\,\mathrm{A})(200\,\Omega) = 35.0\,\mathrm{V},$$

$$V_L = IX_L = (0.i75\,\mathrm{A})(86.7\,\Omega) = 15.2\,\mathrm{V},$$

and

$$V_C = IX_C = (0.175\,\mathrm{A})(37.9\,\Omega) = 6,63\,\mathrm{V}.$$

Note that $X_L > X_C$, so that $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale on the right.



45

(a) For a given amplitude $\mathcal{E}_m$ of the generator emf, the current amplitude is given by

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\,,$$

where R is the resistance, L is the inductance, C is the capacitance, and ω_d is the angular frequency. To find the maximum, set the derivative with respect to ω_d equal to zero and solve for ω_d. The derivative is

$$\frac{dI}{d\omega_d} = -\mathcal{E}_m \left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right] \left[L + \frac{1}{\omega_d^2 C}\right] .$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$ and it does for $\omega_d = 1/\sqrt{LC}$. For the given circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}} = 224\,\mathrm{rad/s} .$$

(b) For this value of the angular frequency, the impedance is $Z = R$ and the current amplitude is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\mathrm{V}}{5.00\,\Omega} = 6.00\,\mathrm{A} .$$

(c) and (d) You want to find the values of ω_d for which $I = \mathcal{E}_m/2R$. This means

$$\frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\mathcal{E}_m}{2R} .$$

Cancel the factors $\mathcal{E}_m$ that appear on both sides, square both sides, and set the reciprocals of the two sides equal to each other to obtain

$$R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 4R^2 .$$

Thus

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2 .$$

Now take the square root of both sides and multiply by $\omega_d C$ to obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3} CR\right) - 1 = 0 ,$$

where the symbol $\pm$ indicates the two possible signs for the square root. The last equation is a quadratic equation for ω_d. Its solutions are

$$\omega_d = \frac{\pm\sqrt{3}CR \pm \sqrt{3C^2R^2 + 4LC}}{2LC} .$$

You want the two positive solutions. The smaller of these is

$$\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219\,\mathrm{rad/s}
\end{aligned}$$

and the larger is

$$\begin{aligned}\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\ &= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\ &\quad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\ &= 228\,\mathrm{rad/s}\,.\end{aligned}$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\mathrm{rad/s} - 219\,\mathrm{rad/s}}{224\,\mathrm{rad/s}} = 0.04\,.$$

<u>49</u>

Use the expressions found in Problem 31–45:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}$$

and

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}\,.$$

Also use

$$\omega = \frac{1}{\sqrt{LC}}\,.$$

Thus

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}\,.$$

<u>55</u>

(a) The impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\,,$$

where R is the resistance, X_L is the inductive reactance, and X_C is the capacitive reactance. Thus

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\,\Omega\,.$$

(b) The average rate at which energy is supplied to the air conditioner is given by

$$P_{\mathrm{avg}} = \frac{\mathcal{E}_{\mathrm{rms}}^2}{Z}\,\cos\phi\,,$$

where $\cos\phi$ is the power factor. Now

$$\cos\phi = \frac{R}{Z} = \frac{12\,\Omega}{12.1\,\Omega} = 0.992\,,$$

so

$$P_{\text{avg}} = \left[\frac{(120\text{ V})^2}{12.1\,\Omega}\right](0.992) = 1.18\times 10^3\text{ W}\,.$$

57

(a) The power factor is $\cos\phi$, where ϕ is the phase angle when the current is written $i = I\sin(\omega_d t - \phi)$. Thus $\phi = -42.0°$ and $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega_d t - \phi > \omega_d t$ and the current leads the emf.

(c) The phase angle is given by $\tan\phi = (X_L - X_C)/R$, where X_L is the inductive reactance, X_C is the capacitive reactance, and R is the resistance. Now $\tan\phi = \tan(-42.0°) = -0.900$, a negative number. This means $X_L - X_C$ is negative, or $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit is in resonance, X_L is the same as X_C, $\tan\phi$ is zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e), (f), and (g) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor. The inductive reactance may be zero, so there need not be an inductor. If there is an inductor, its reactance must be less than that of the capacitor at the operating frequency.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\mathcal{E}_m I\cos\phi = \frac{1}{2}(75.0\text{ V})(1.20\text{ A})(0.743) = 33.4\text{ W}\,.$$

(i) The answers above depend on the frequency only through the phase angle ϕ, which is given. If values are given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

63

(a) If N_p is the number of primary turns and N_s is the number of secondary turns, then

$$V_s = \frac{N_s}{N_p}V_p = \left(\frac{10}{500}\right)(120\text{ V}) = 2.4\text{ V}\,.$$

(b) and (c) The current in the secondary is given by Ohm's law:

$$I_s = \frac{V_s}{R_s} = \frac{2.4\text{ V}}{15\,\Omega} = 0.16\text{ A}\,.$$

The current in the primary is

$$I_p = \frac{N_s}{N_p}I_s = \left(\frac{10}{500}\right)(0.16\text{ A}) = 3.2\times 10^{-3}\text{ A}\,.$$

67

Use the trigonometric identity, found in Appendix E,

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right),$$

where α and β are any two angles. Thus

$$V_1 - V_2 = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin(120°)\cos(\omega_d t - 60°) = \sqrt{3}A\cos(\omega_d t - 60°),$$

where $\sin(120°) = \sqrt{3}/2$ was used. Similarly,

$$V_1 - V_3 = A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin(240°)\cos(\omega_d t - 120°) = -\sqrt{3}A\cos(\omega_d t - 120°),$$

where $\sin(240°) = -\sqrt{3}/2$ was used, and

$$\begin{aligned} V_2 - V_3 &= A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin(120°)\cos(\omega_d t - 180°) \\ &= \sqrt{3}A\cos(\omega_d t - 180°). \end{aligned}$$

All of these are sinusoidal functions of ω_d and all have amplitudes of $\sqrt{3}A$.

71

(a) Let V_C be the maximum potential difference across the capacitor, V_L be the maximum potential difference across the inductor, and V_R be the maximum potential difference across the resistor. Then the phase constant ϕ is

$$\tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{2.00V_R - V_R}{V_R}\right) = \tan^{-1}(1.00) = 45.0°.$$

(b) Since the maximum emf is related to the current amplitude by $\mathcal{E}_m = IZ$, where Z is the impedance and $R = Z\cos\phi$,

$$R = \frac{\mathcal{E}_m\cos\phi}{I} = \frac{(30.0\,\text{V})\cos 45°}{300\times10^{-3}\,\text{A}} = 70.7\,\Omega.$$

73

(a) The frequency of oscillation of an LC circuit is $f = 1/2\pi\sqrt{LC}$, where L is the inductance and C is the capacitance. Thus

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10.4\times10^3\,\text{Hz})^2(340\times10^{-6}\,\text{F})} = 6.89\times10^{-7}\,\text{H}.$$

(b) The total energy is $U = \frac{1}{2}LI^2$, where I is the current amplitude. Thus $U = \frac{1}{2}(6.89\times10^{-7}\,\text{H})(7.20\times10^{-3}\,\text{A})^2 = 1.79\times10^{-11}\,\text{J}$.

(c) The total energy is also given by $U = Q^2/2C$, where Q is the charge amplitude. Thus $Q = \sqrt{2UC} = \sqrt{2(1.79 \times 10^{-11}\,\text{J})(340 \times 10^{-6}\,\text{F})} = 1.10 \times 10^{-7}\,\text{C}$.

83

(a) The total energy U of the circuit is the sum of the energy U_E stored in the capacitor and the energy U_B stored in the inductor at the same time. Since $U_B = 2.00U_E$, the total energy is $U = 3.00U_E$. Now $U = Q^2/2C$ and $U_E = q^2/2C$, where Q is the maximum charge, q is the charge when the magnetic energy is twice the electrical energy, and C is the capacitance. Thus $Q^2/2C = 3.00q^2/2C$ and $q = Q/\sqrt{3.00} = 0.577Q$.

(b) If the capacitor has maximum charge at time $t = 0$, then $q = Q\cos(\omega t)$, where ω is the angular frequency of oscillation. This means $\omega t = \cos^{-1}(0.577) = 0.964\,\text{rad}$. Since $\omega = 2\pi/T$, where T is the period,

$$t = \frac{0.964}{2\pi}T = 0.153T\,.$$

85

(a) The energy stored in a capacitor is given by $U_E = q^2/2C$, where q is the charge and C is the capacitance. Now q^2 is periodic with a period of $T/2$, where T is the period of the driving emf, so U_E has the same value at the beginning and end of each cycle. Actually U_E has the same value at the beginning and end of each half cycle.

(b) The energy stored in an inductor is given by $Li^2/2$, where i is the current and L is the inductance. The square of the current is periodic with a period of $T/2$, so it has the same value at the beginning and end of each cycle.

(c) The rate with which the driving emf device supplies energy is

$$P_\mathcal{E} = i\mathcal{E} = I\mathcal{E}_m \sin(\omega_d t)\sin(\omega_d t - \phi)\,,$$

where I is the current amplitude, $\mathcal{E}_m$ is the emf amplitude, ω is the angular frequency, and ϕ is a phase constant. The energy supplied over a cycle is

$$\begin{aligned} E_\mathcal{E} &= \int_0^T P_\mathcal{E}\,dt = I\mathcal{E}_m \int_0^T \sin(\omega_d t)\sin(\omega_d t - \phi)\,dt \\ &= I\mathcal{E}_m \int_0^T \sin(\omega_d t)\left[\sin(\omega_d t)cos(\phi) - \cos(\omega_d t)\sin(\phi)\right]\,dt\,, \end{aligned}$$

where the trigonometric identity $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ was used. Now the integral of $\sin^2(\omega_d t)$ over a cycle is $T/2$ and the integral of $\sin(\omega_d t)\cos(\omega_d t)$ over a cycle is zero, so $E_\mathcal{E} = \frac{1}{2}I\mathcal{E}_m\cos\phi$.

(d) The rate of energy dissipation in a resistor is given by

$$P_R = i^2R = I^2\sin^2(\omega_d t - \phi)$$

and the energy dissipated over a cycle is

$$E_R = I^2\int_0^T \sin^2(\omega_d t - \phi)\,dt = \tfrac{1}{2}I^2RT\,.$$

(e) Now $\mathcal{E}_m = IZ$, where Z is the impedance, and $R = Z\cos\phi$, so $E_\mathcal{E} = \frac{1}{2}I^2TZ\cos\phi = \frac{1}{2}I^2RT = E_R$.

Chapter 32

3

(a) Use Gauss' law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$. Write $\oint \vec{B} \cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C$, where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end, the magnetic field is inward, so the flux is $\Phi_1 = -25.0\,\mu\text{Wb}$. Over the second end, the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. Its value is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60 \times 10^{-3}\,\text{T}) = +7.24 \times 10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}\,.$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\,.$$

(b) The minus sign indicates that the flux is inward through the curved surface.

5

Consider a circle of radius r (= 6.0 mm), between the plates and with its center on the axis of the capacitor. The current through this circle is zero, so the Ampere-Maxwell law becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}\,,$$

where $\vec{B}$ is the magnetic field at points on the circle and Φ_E is the electric flux through the circle. The magnetic field is tangent to the circle at all points on it, so $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. The electric flux through the circle is $\Phi_E = \pi R^2 E$, where R (= 3.0 mm) is the radius of a capacitor plate. When these substitutions are made, the Ampere-Maxwell law becomes

$$2\pi r B = \mu_0 \epsilon_0 \pi R^2 \frac{dE}{dt}\,.$$

Thus

$$\frac{dE}{dt} = \frac{2rB}{\mu_0 \epsilon_0 R^2} = \frac{2(6.0 \times 10^{-3}\,\text{m})(2.0 \times 10^{-7}\,\text{T})}{(4\pi \times 10^{-7}\,\text{H/m})(8.85 \times 10^{-12}\,\text{Fm})(3.0 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{13}\,\text{V/m} \cdot \text{s}\,.$$

13

The displacement current is given by

$$i_d = \epsilon_0 A \frac{dE}{dt}\,,$$

where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus

$$i_d = \frac{\epsilon_0 A}{d}\frac{dV}{dt}.$$

Now $\epsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor without a dielectric, so

$$i_d = C\frac{dV}{dt}.$$

<u>21</u>

(a) For a parallel-plate capacitor, the charge q on the positive plate is given by $q = (\epsilon_0 A/d)V$, where A is the plate area, d is the plate separation, and V is the potential difference between the plates. In terms of the electric field E between the plates, $V = Ed$, so $q = \epsilon_0 AE = \epsilon_0 \Phi_E$, where Φ_E is the total electric flux through the region between the plates. The true current into the positive plate is $i = dq/dt = \epsilon_0\, d\Phi_E/dt = i_{d\text{ total}}$, where $i_{d\text{ total}}$ is the total displacement current between the plates. Thus $i_{d\text{ total}} = 2.0\,\text{A}$.

(b) Since $i_{d\text{ total}} = \epsilon_0\, d\Phi_E/dt = \epsilon_0 A\, dE/dt$,

$$\frac{dE}{dt} = \frac{i_{d\text{ total}}}{\epsilon_0 A} = \frac{2.0\,\text{A}}{(8.85\times10^{-12}\,\text{F/m})(1.0\,\text{m})^2} = 2.3\times10^{11}\,\text{V/m}\cdot\text{s}.$$

(c) The displacement current is uniformly distributed over the area. If a is the area enclosed by the dashed lines and A is the area of a plate, then the displacement current through the dashed path is

$$i_{d\text{ enc}} = \frac{a}{A} i_{d\text{ total}} = \frac{(0.50\,\text{m})^2}{(1.0\,\text{m})^2}(2.0\,\text{A}) = 0.50\,\text{A}.$$

(d) According to Maxwell's law of induction,

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{d\text{ enc}} = (4\pi\times10^{-7}\,\text{H/m})(0.50\,\text{A}) = 6.3\times10^{-7}\,\text{T}\cdot\text{m}.$$

Notice that the integral is around the dashed path and the displacement current on the right side of the Maxwell's law equation is the displacement current through that path, not the total displacement current.

<u>35</u>

(a) The z component of the orbital angular momentum is given by $L_{\text{orb},z} = m_\ell h/2\pi$, where h is the Planck constant. Since $m_\ell = 0$, $L_{\text{orb},z} = 0$.

(b) The z component of the orbital contribution to the magnetic dipole moment is given by $\mu_{\text{orb},z} = -m_\ell \mu_B$, where μ_B is the Bohr magneton. Since $m_\ell = 0$, $\mu_{\text{orb},z} = 0$.

(c) The potential energy associated with the orbital contribution to the magnetic dipole moment is given by $U = -\mu_{\text{orb},z} B_{\text{ext}}$, where B_{ext} is the z component of the external magnetic field. Since $\mu_{\text{orb},z} = 0$, $U = 0$.

(d) The z component of the spin magnetic dipole moment is either $+\mu_B$ or $-\mu_B$, so the potential energy is either

$$U = -\mu_B B_{\text{ext}} = -(9.27 \times 10^{-24}\ \text{J/T})(35 \times 10^{-3}\ \text{T}) = -3.2 \times 10^{-25}\ \text{J}\,.$$

or $U = +3.2 \times 10^{-25}$ J.

(e) Substitute m_ℓ into the equations given above. The z component of the orbital angular momentum is

$$L_{\text{orb},\,z} = \frac{m_\ell h}{2\pi} = \frac{(-3)(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi} = -3.2 \times 10^{-34}\ \text{J}\cdot\text{s}\,.$$

(f) The z component of the orbital contribution to the magnetic dipole moment is

$$\mu_{\text{orb},\,z} = -m_\ell \mu_B = -(-3)(9.27 \times 10^{-24}\ \text{J/T}) = 2.8 \times 10^{-23}\ \text{J/T}\,.$$

(g) The potential energy associated with the orbital contribution to the magnetic dipole moment is

$$U = -\mu_{\text{orb},\ z} B_{\text{ext}} = -(2.78 \times 10^{-23}\ \text{J/T})(35 \times 10^{-3}\ \text{T}) = -9.7 \times 10^{-25}\ \text{J}\,.$$

(h) The potential energy associated with spin does not depend on m_ℓ. It is $\pm 3.2 \times 10^{-25}$ J.

39

The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = MV$, where M is the magnetization and V is the volume of the cylinder. Use $V = \pi r^2 L$, where r is the radius of the cylinder and L is its length. Thus

$$\mu = M\pi r^2 L = (5.30 \times 10^3\ \text{A/m})\pi(0.500 \times 10^{-2}\ \text{m})^2(5.00 \times 10^{-2}\ \text{m}) = 2.08 \times 10^{-2}\ \text{J/T}\,.$$

45

(a) The number of atoms per unit volume in states with the dipole moment aligned with the magnetic field is $N_+ = Ae^{\mu B/kT}$ and the number per unit volume in states with the dipole moment antialigned is $N_- = Ae^{-\mu B/kT}$, where A is a constant of proportionality. The total number of atoms per unit volume is $N = N_+ + N_- = A\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)$. Thus

$$A = \frac{N}{e^{\mu B/kT} + e^{-\mu B/kT}}\,.$$

The magnetization is the net dipole moment per unit volume. Subtract the magnitude of the total dipole moment per unit volume of the antialigned moments from the total dipole moment per unit volume of the aligned moments. The result is

$$M = \frac{N\mu e^{\mu B/kT} - N\mu e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \frac{N\mu\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{e^{\mu B/kT} + e^{-\mu B/kT}} = N\mu \tanh(\mu B/kT)\,.$$

(b) If $\mu B \ll kT$, then $e^{\mu B/kT} \approx 1 + \mu B/kT$ and $e^{-\mu B/kT} \approx 1 - \mu B/kT$. (See Appendix E for the power series expansion of the exponential function.) The expression for the magnetization becomes

$$M \approx \frac{N\mu\left[(1+\mu B/kT) - (1-\mu B/kT)\right]}{(1+\mu B/kT) + (1-\mu B/kT)} = \frac{N\mu^2 B}{kT}\,.$$

(c) If $\mu B \gg kT$, then $e^{-\mu B/kT}$ is negligible compared to $e^{\mu B/kT}$ in both the numerator and denominator of the expression for M. Thus

$$M \approx \frac{N\mu e^{\mu B/kT}}{e^{\mu B/kT}} = N\mu\,.$$

(d) The expression for M predicts that it is linear in B/kT for $\mu B/kT$ small and independent of B/kT for $\mu B/kT$ large. The figure agrees with these predictions.

47

(a) The field of a dipole along its axis is given by Eq. 29–27:

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}\,,$$

where μ is the dipole moment and z is the distance from the dipole. Thus the magnitude of the magnetic field is

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.5 \times 10^{-23}\,\mathrm{J/T})}{2\pi(10 \times 10^{-9}\,\mathrm{m})^3} = 3.0 \times 10^{-6}\,\mathrm{T}\,.$$

(b) The energy of a magnetic dipole with dipole moment $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, where ϕ is the angle between the dipole moment and the field. The energy required to turn it end for end (from $\phi = 0°$ to $\phi = 180°$) is

$$\begin{aligned}\Delta U &= -\mu B(\cos 180° - \cos 0°) = 2\mu B = 2(1.5 \times 10^{-23}\,\mathrm{J/T})(3.0 \times 10^{-6}\,\mathrm{T}) \\ &= 9.0 \times 10^{-29}\,\mathrm{J} = 5.6 \times 10^{-10}\,\mathrm{eV}\,.\end{aligned}$$

The mean kinetic energy of translation at room temperature is about 0.04 eV (see Eq. 19–24 or Sample Problem 32–3). Thus if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

53

(a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\,.$$

Substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3 \mu}{3m}.$$

Solve for R and obtain

$$R = \left[\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right]^{1/3}.$$

The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66 \times 10^{-27}\,\text{kg/u}) = 9.30 \times 10^{-26}\,\text{kg}.$$

So

$$R = \left[\frac{3(9.30 \times 10^{-26}\,\text{kg})(8.0 \times 10^{22}\,\text{J/T})}{4\pi(14 \times 10^3\,\text{kg/m}^3)(2.1 \times 10^{-23}\,\text{J/T})}\right]^{1/3} = 1.8 \times 10^5\,\text{m}.$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82 \times 10^5\,\text{m})^3 = 2.53 \times 10^{16}\,\text{m}^3$$

and the volume of Earth is

$$V_e = \frac{4\pi}{3}(6.37 \times 10^6\,\text{m})^3 = 1.08 \times 10^{21}\,\text{m}^3,$$

so the fraction of Earth's volume that is occupied by the sphere is

$$\frac{2.53 \times 10^{16}\,\text{m}^3}{1.08 \times 10^{21}\,\text{m}^3} = 2.3 \times 10^{-5}.$$

The radius of Earth was obtained from Appendix C.

55

(a) The horizontal and vertical directions are perpendicular to each other, so the magnitude of the field is

$$B = \sqrt{B_h^2 + B_v^2} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 - \sin^2\lambda_m + 4\sin^2\lambda_m}$$
$$= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2\lambda_m},$$

where the trigonometric identity $\cos^2\lambda_m = 1 - \sin^2\lambda_m$ was used.

(b) The tangent of the inclination angle is

$$\tan\phi_i = \frac{B_v}{B_h} = \left(\frac{\mu_0\mu}{2\pi r^3 \sin\lambda_m}\right)\left(\frac{4\pi r^3}{\mu_0\mu\cos\lambda_m} = \frac{2\sin\lambda_m}{\cos\lambda_m}\right) = 2\tan\lambda_m,$$

where $\tan\lambda_m = (\sin\lambda_m)/(\cos\lambda_m)$ was used.

61

(a) The z component of the orbital angular momentum can have the values $L_{\text{orb},z} = m_\ell h/2\pi$, where m_ℓ can take on any integer value from -3 to $+3$, inclusive. There are seven such values (-3, -2, -1, 0, +1, +2, and +3).

(b) The z component of the orbital magnetic moment is given by $\mu_{\text{orb},z} = -m_\ell eh/4\pi m$, where m is the electron mass. Since there is a different value for each possible value of m_ℓ, there are seven different values in all.

(c) The greatest possible value of $L_{\text{orb},z}$ occurs if $m_\ell = +3$ is $3h/2\pi$.

(d) The greatest value of $\mu_{\text{orb},\,z}$ is $3eh/4\pi m$.

(e) Add the orbital and spin angular momenta: $L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = (m_\ell h/2\pi) + (m_s h/2\pi)$. To obtain the maximum value, set m_ℓ equal to +3 and m_s equal to $+\frac{1}{2}$. The result is $L_{\text{net},z} = 3.5h/2\pi$.

(f) Write $L_{\text{net},z} = Mh/2\pi$, where M is half an odd integer. M can take on all such values from -3.5 to $+3.5$. There are eight of these: -3.5, -2.5, -1.5, -0.5, $+0.5$, $+1.5$, $+2.5$, and $+3.5$.

Chapter 33

5

If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. The frequency is the same as the frequency of oscillation of the current in the LC circuit of the generator. That is, $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c\,.$$

The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{(550\times10^{-9}\,\mathrm{m})^2}{4\pi^2(17\times10^{-12}\,\mathrm{F})(3.00\times10^{8}\,\mathrm{m/s})^2} = 5.00\times10^{-21}\,\mathrm{H}\,.$$

This is exceedingly small.

21

The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity. The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation. Thus

$$p_r = \frac{2P}{Ac} = \frac{2(1.5\times10^{9}\,\mathrm{W})}{(1.00\times10^{-6}\,\mathrm{m}^2)(3.00\times10^{8}\,\mathrm{m/s})} = 1.0\times10^{7}\,\mathrm{Pa} = 10\,\mathrm{MPa}\,.$$

23

Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1-f$. The reflected portion exerts a radiation pressure of $p_r = (2fI_0)/c$ and the absorbed portion exerts a radiation pressure of $p_a = (1-f)I_0/c$, where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c}\,.$$

To relate the intensity and energy density, consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy will pass through the end in time $t = \ell/c$ so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc\,.$$

Thus $u = I/c$. The intensity and energy density are inherently positive, regardless of the propagation direction.

For the partially reflected and partially absorbed wave, the intensity just outside the surface is $I = I_0 + fI_0 = (1+f)I_0$, where the first term is associated with the incident beam and the second is associated with the reflected beam. The energy density is, therefore,

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c} ,$$

the same as radiation pressure.

25

(a) Since $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{3.0\,\mathrm{m}} = 1.0 \times 10^8\,\mathrm{Hz} .$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0 \times 10^8\,\mathrm{Hz}) = 6.3 \times 10^8\,\mathrm{rad/s} .$$

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\mathrm{m}} = 2.1\,\mathrm{rad/m} .$$

(d) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\mathrm{V/m}}{3.00 \times 10^8\,\mathrm{m/s}} = 1.00 \times 10^{-6}\,\mathrm{T} .$$

(e) $\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E} \times \vec{B}$ to be in the positive x direction (the direction of propagation).

(f) The time-averaged rate of energy flow or intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{H/m})(3.00 \times 10^8\,\mathrm{m/s})} = 1.2 \times 10^2\,\mathrm{W/m^2} .$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\,\mathrm{W/m^2})(2.0\,\mathrm{m^2})}{3.00 \times 10^8\,\mathrm{m/s}} = 8.0 \times 10^{-7}\,\mathrm{N} .$$

(h) The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0 \times 10^{-7}\,\mathrm{N}}{2.0\,\mathrm{m^2}} = 4.0 \times 10^{-7}\,\mathrm{Pa} .$$

27

If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. Thus $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10 \times 10^3\,\mathrm{W})(1\,d)(8.64 \times 10^4\,\mathrm{s/d})}{(1.5 \times 10^3\,\mathrm{kg})(3.00 \times 10^8\,\mathrm{m/s})} = 1.9 \times 10^{-3}\,\mathrm{m/s} = 1.9\,\mathrm{mm/s}\,.$$

35

Let I_0 be in the intensity of the unpolarized light that is incident on the first polarizing sheet. Then the transmitted intensity is $I_1 = \frac{1}{2}I_0$ and the direction of polarization of the transmitted light is θ_1 (= 40°) counterclockwise from the y axis in the diagram.

The polarizing direction of the second sheet is θ_2 (= 20°) clockwise from the y axis so the angle between the direction of polarization of the light that is incident on that sheet and the polarizing direction of the of the sheet is 40° + 20° = 60°. The transmitted intensity is

$$I_2 = I_1 \cos^2 60° = \frac{1}{2} I_0 \cos^2 60°$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis.

The polarizing direction of the third sheet is θ_3 (= 40°) counterclockwise from the y axis so the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60° = \frac{1}{2} I_0 \cos^4 60° = 3.1 \times 10^{-2}\,.$$

3.1% of the light's initial intensity is transmitted.

43

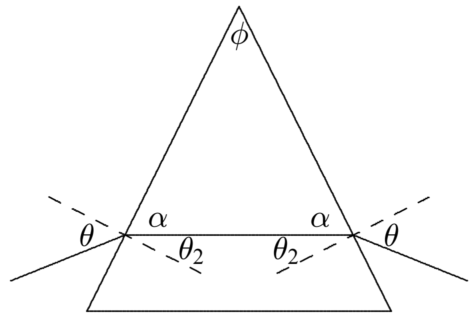
(a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

It can be done with two sheets. Place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

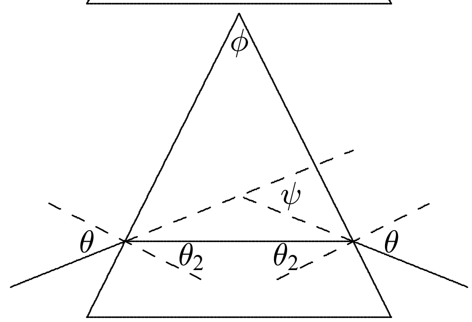
(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ with the direction of polarization of the incident radiation and with the polarizing direction of each successive sheet rotated $90°/n$ in the same direction from the polarizing direction of the previous sheet. The transmitted radiation is polarized with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is $I = I_0 \cos^{2n}(90°/n)$. You want the smallest integer value of n for which this is greater than $0.60 I_0$.

Start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, you have obtained the solution. If it is less, increase n by 1 and try again. Repeat this process, increasing n by 1 each time, until you have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

55



Look at the diagram on the right. The two angles labeled α have the same value. θ_2 is the angle of refraction. Because the dotted lines are perpendicular to the prism surface $\theta_2 + \alpha = 90°$ and $\alpha = 90° - \theta_2$. Because the interior angles of a triangle sum to 180°, $180° - 2\theta_2 + \phi = 180°$ and $\theta_2 = \phi/2$.



Now look at the next diagram and consider the triangle formed by the two normals and the ray in the interior. The two equal interior angles each have the value $\theta - \theta_2$. Because the exterior angle of a triangle is equal to the sum of the two opposite interior angles, $\psi = 2(\theta - \theta_2)$ and $\theta = \theta_2 + \psi/2$. Upon substitution for θ_2 this becomes $\theta = (\phi + \psi)/2$.

According to the law of refraction the index of refraction of the prism material is

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin(\phi+\psi)/2}{\sin\phi/2}\,.$$

65

(a) No refraction occurs at the surface ab, so the angle of incidence at surface ac is $90° - \phi$. For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, you want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°\,.$$

The index of refraction for air was taken to be unity.

(b) Replace the air with water. If n_w (= 1.33) is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°\,.$$

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The angle of incidence θ_B for which reflected light is fully polarized is given by Eq. 33–49 of the text. If n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium, then $\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 63.8°$.

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Let θ_1 (= 45°) be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n \sin\theta_3 \geq 1$. You want to find the smallest value of the index of refraction n for which this inequality holds.

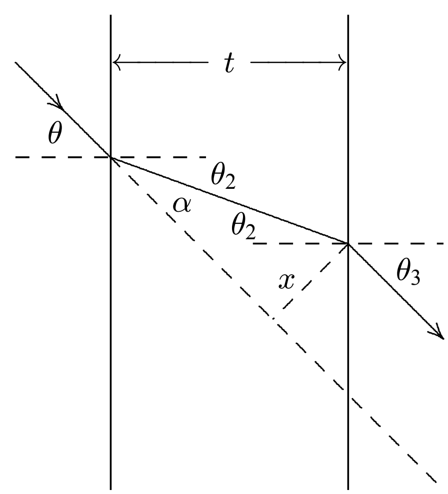
The law of refraction, applied to the first surface, yields $n \sin\theta_2 = \sin\theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus the condition for total internal reflection becomes $1 \leq n \sin(90° - \theta_2) = n \cos\theta_2$. Square this equation and use $\sin^2\theta_2 + \cos^2\theta_2 = 1$ to obtain $1 \leq n^2(1 - \sin^2\theta_2)$. Now substitute $\sin\theta_2 = (1/n)\sin\theta_1$ to obtain

$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1 \,.$$

The largest value of n for which this equation is true is the value for which $1 = n^2 - \sin^2\theta_1$. Solve for n:

$$n = \sqrt{1 + \sin^2\theta_1} = \sqrt{1 + \sin^2 45°} = 1.22 \,.$$

75



Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields $\sin\theta = n\sin\theta_2$. The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then $n\sin\theta_2 = \sin\theta_3$. Thus $\sin\theta_3 = \sin\theta$ and $\theta_3 = \theta$. The emerging ray is parallel to the incident ray.

You wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D\cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and $x = D\sin\alpha = D\sin(\theta - \theta_2)$. Thus

$$x = \frac{t\sin(\theta - \theta_2)}{\cos\theta_2} \,.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}\,.$$

<u>77</u>

The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

(a) Take d to be $150\,\text{km} = 150 \times 10^3$ m. Then,

$$t = \frac{d}{c} = \frac{150 \times 10^3\,\text{m}}{3.00 \times 10^8\,\text{m/s}} = 5.00 \times 10^{-4}\,\text{s}\,.$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is $d = (1.5 \times 10^8\,\text{km}) + 2(3.8 \times 10^5\,\text{km}) = 1.51 \times 10^8\,\text{km} = 1.51 \times 10^{11}$ m. The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51 \times 10^{11}\,\text{m}}{3.00 \times 10^8\,\text{m/s}} = 500\,\text{s} = 8.4\,\text{min}\,.$$

The distances are given in the problem.

(c) Take d to be $2(1.3 \times 10^9\,\text{km}) = 2.6 \times 10^{12}$ m. Then,

$$t = \frac{d}{c} = \frac{2.6 \times 10^{12}\,\text{m}}{3.00 \times 10^8\,\text{m/s}} = 8.7 \times 10^3\,\text{s} = 2.4\,\text{h}\,.$$

(d) Take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500\,\text{ly}}{1.00\,\text{ly/y}} = 6500\,\text{y}\,.$$

The explosion took place in the year $1054 - 6500 = -5446$ or B.C. 5446.

<u>79</u>

(a) The amplitude of the magnetic field is $B = E/c = (5.00\,\text{V/m})/(3.00 \times 10^8\,\text{m/s}) = 1.67 \times 10^{-8}$ T. According to the argument of the trigonometric function in the expression for the electric field, the wave is moving in the negative z direction and the electric field is parallel to the y axis. In order for $\vec{E} \times \vec{B}$ to be in the negative z direction, $\vec{B}$ must be in the positive x direction when $\vec{E}$ is in the positive y direction. Thus

$$B_x = (1.67 \times 10^{-8}\,\text{T})\sin[(1.00 \times 10^6\,\text{m}^{-1})z + \omega t]$$

is the only nonvanishing component of the magnetic field.

The angular wave number is $k = 1.00 \times 10^6\,\text{m}^{-1}$ so the angular frequency is $\omega = kc = (1.00 \times 10^6\,\text{m}^{-1})(3.00 \times 10^8\,\text{m/s}) = 3.00 \times 10^{14}\,\text{s}^{-1}$ and

$$B_x = (1.67 \times 10^{-8}\,\text{T})\sin[(1.00 \times 10^6\,\text{m}^{-1})z + (3.00 \times 10^{14}\,\text{s}^{-1})t]\,.$$

(b) The wavelength is $\lambda = 2\pi/k = 2\pi/(1.00 \times 10^6\,\text{m}^{-1}) = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2\pi/(3.00 \times 10^{14}\,\mathrm{s}^{-1}) = 2.09 \times 10^{-14}\,\mathrm{s}$.

(d) The intensity of this wave is $I = E_m^2/2\mu_0 c = (5.00\,\mathrm{V/m})^2/2(4\pi\times10^{-7}\,\mathrm{H/m})(3.00\times10^8\,\mathrm{m/s} = 0.0332\,\mathrm{W/m^2}$. (f) A wavelength of $6.28 \times 10^{-6}\,\mathrm{m}$ places this wave in the infrared portion of the electromagnetic spectrum. See Fig. 33-1.

83

(a) The power is the same through any hemisphere centered at the source. The area of a hemisphere of radius r is $A = 2\pi r^2$. In this case r is the distance from the source to the aircraft. Thus the intensity at the aircraft is $I = P/A = P/2\pi r^2 = (180 \times 10^3\,\mathrm{W})/2\pi(90 \times 10^3\,\mathrm{m})^2 = 3.5 \times 10^{-6}\,\mathrm{W/m^2}$.

(b) The power of the reflection is the product of the intensity at the aircraft and the cross section of the aircraft: $P_r = (3.5 \times 10^{-6}\,\mathrm{W/m^2})(0.22\,\mathrm{m^2}) = 7.8 \times 10^{-7}\,\mathrm{W}$.

(c) The intensity at the detector is $P_r/2\pi r^2 = (7.8 \times 10^{-7}\,\mathrm{W})/2\pi(90 \times 10^3\,\mathrm{m})^2 = 1.5 \times 10^{-17}\,\mathrm{W/m^2}$.

(d) Since the intensity is given by $I = E_m^2/2\mu_0 c$,

$$E_m = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi \times 10^{-7}\,\mathrm{H/m})(3.00 \times 10^8\,\mathrm{m/s})(1.5 \times 10^{-17}\,\mathrm{W/m^2})} = 1.1 \times 10^{-7}\,\mathrm{V/m}\,.$$

(e) The rms value of the magnetic field is $B_{\mathrm{rms}} = E_m/\sqrt{2}c = (1.1 \times 10^{-7}\,\mathrm{V/m})/(\sqrt{2})(3.00 \times 10^8\,\mathrm{m/s}) = 2.5 \times 10^{-16}\,\mathrm{T}$.

91

The critical angle for total internal reflection is given by $\theta_c = \sin^{-1}(1/n)$. For $n = 1.456$ this angle is $\theta_c = 43.38°$ and for $n = 1.470$ it is $\theta_c = 42.86°$.

(a) An incidence angle of 42.00° is less than the critical angle for both red and blue light. The refracted light is white.

(b) An incidence angle of 43.10° is less than the critical angle for red light and greater than the critical angle for blue light. Red light is refracted but blue light is not. The refracted light is reddish.

(c) An incidence angle of 44.00° is greater than the critical angle for both red and blue light. Neither is refracted.

103

(a) Take the derivative of the functions given for E and B, then substitute them into

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2} \qquad \text{and} \qquad \frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\,.$$

The derivatives of E are $\partial^2 E/\partial t^2 = -\omega^2 E_m \sin(kx - \omega t)$ and $\partial^2 E/\partial x^2 = -k^2 E_m \sin(kx - \omega t)$, so the wave equation for the electric field yields $\omega^2 = c^2 k^2$. Since $\omega = ck$ the function satisfies the wave equation. Similarly, the derivatives of B are $\partial^2 B/\partial t^2 = -\omega^2 B_m \sin(kx - \omega t)$ and $\partial^2 B/\partial x^2 = -k^2 B_m \sin(kx - \omega t)$ and the wave equation for the magnetic field yields $\omega^2 = c^2 k^2$. Since $\omega = ck$ the function satisfies the wave equation.

(b) Let $u = kx \pm \omega t$ and consider f to be a function of u, which in turn is a function of x and t. Then the chain rule of the calculus gives

$$\frac{\partial^2 E}{\partial t^2} = \frac{d^2 f}{du^2}\left(\frac{\partial u}{\partial t}\right)^2 = \frac{d^2 f}{du^2}\omega^2$$

and

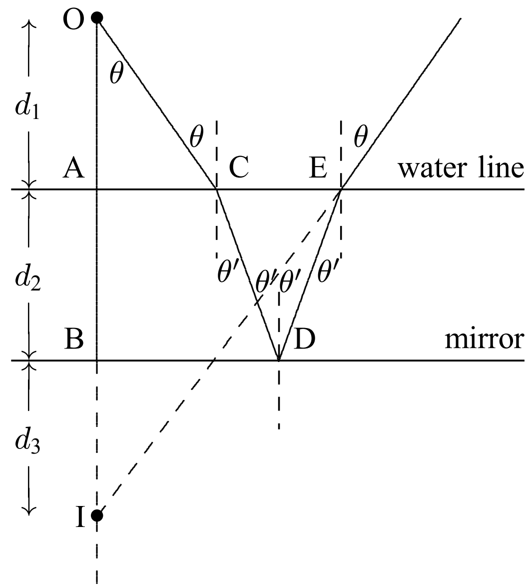
$$\frac{\partial^2 E}{\partial x^2} = \frac{d^2 f}{du^2}\left(\frac{\partial u}{\partial x}\right)^2 = \frac{d^2 f}{du^2}k^2 .$$

Substitution into the wave equation again yields $\omega^2 = c^2k^2$, so the function obeys the wave equation. A similar analysis shows that the function for B also satisfies the wave equation.

Chapter 34

5

The light bulb is labeled O and its image is labeled I on the digram to the right. Consider the two rays shown on the diagram to the right. One enters the water at A and is reflected from the mirror at B. This ray is perpendicular to the water line and mirror. The second ray leaves the lightbulb at the angle θ, enters the water at C, where it is refracted. It is reflected from the mirror at D and leaves the water at E. At C the angle of incidence is θ and the angle of refraction is θ'. At D the angles of incidence and reflection are both θ'. At E the angle of incidence is θ' and the angle of refraction is θ. The dotted lines that meet at I represent extensions of the emerging rays. Light appears to come from I. We want to compute d_3.



Consideration of the triangle OBE tells us that the distance $d_2 + d_3$ is $L\tan(90° - \theta) = L/\tan\theta$, where L is the distance between A and E. Consideration of the triangle OBC tells us that the distance between A and C is $d_1 \tan\theta$ and consideration of the triangle CDE tells us that the distance between C and E is $2d_2 \tan\theta'$, so $L = d_1 \tan\theta + 2d_2 \tan\theta'$, $d_2 + d_3 = (d_1 \tan\theta + 2d_2 \tan\theta')/\tan\theta$, and

$$d_3 = \frac{d_1 \tan\theta + 2d_2 \tan\theta'}{\tan\theta} - d_2 .$$

Apply the law of refraction at point C: $\sin\theta = n \sin\theta'$, where n is the index of refraction of water. Since the angles θ and θ' are small we may approximate their sines by their tangents and write $\tan\theta = n \tan\theta'$. Us this to substitute for $\tan\theta$ in the expression for d_3 to obtain

$$d_3 = \frac{nd_1 + 2d_2}{n} - d_2 = \frac{(1.33)(250\,\text{cm}) + 2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 350\,\text{cm}\,,$$

where the index of refraction of water was taken to be 1.33.

11

(a) The radius of curvature r and focal length f are positive for a concave mirror and are related by $f = r/2$, so $r = 2(+12\,\text{cm}) = +24\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(12\,\text{cm})(18\,\text{cm})}{18\,\text{cm} - 12\,\text{cm}} = 36\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(36\,\text{cm})/(18\,\text{cm} = -2.0$.

(d) The value obtained for i is positive, so the image is real.

(e) The value obtained for the magnification is negative, so the image is inverted.

(f) Real images are formed by mirrors on the same side as the object. Since the image here is real it is on the same side of the mirror as the object.

<u>**9**</u>

(a) The radius of curvature r and focal length f are positive for a concave mirror and are related by $f = r/2$, so $r = 2(+18\,\text{cm}) = +36\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(18\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 18\,\text{cm}} = -36\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(-36\,\text{cm})/(12\,\text{cm} = 3.0$.

(d) The value obtained for i is negative, so the image is virtual.

(e) The value obtained for the magnification is positive, so the image is not inverted.

(f) Real images are formed by mirrors on the same side as the object and virtual images are formed on the opposite side. Since the image here is virtual it is on the opposite side of the mirror from the object.

<u>**15**</u>

(a) The radius of curvature r and focal length f are negative for a convex mirror and are related by $f = r/2$, so $r = 2(-10\,\text{cm}) = -20\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(-10\,\text{cm})(8\,\text{cm})}{(8\,\text{cm}) - (-10\,\text{cm})} = -4.44\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(-4.44\,\text{cm})/(8\,\text{cm} = +0.56$.

(d) The value obtained for i is negative, so the image is virtual.

(e) The value obtained for the magnification is positive, so the image is not inverted.

(f) Real images are formed by mirrors on the same side as the object and virtual images are formed on the opposite side. Since the image here is virtual it is on the opposite side of the mirror from the object

<u>**27**</u>

Since the mirror is convex the radius of curvature is negative. The focal length is $f = r/2 = (-40\,\text{cm})/2 = -20\,\text{cm}$.

Since $(1/p) + (1/i) = (1/f)$,

$$p = \frac{if}{i - f}.$$

This yields $p = +5.0$ cm if $i = -4.0$ cm and $p = -3.3$ cm if $i = -4.0$ cm. Since p must be positive we select $i = -4.0$ cm and take p to be $+5.0$ cm.

The magnification is $m = -i/p = -(-4.0\,\text{cm})/(5.0\,\text{cm}) = +0.80$. Since the image distance is negative the image is virtual and on the opposite side of the mirror from the object. Since the magnification is positive the image is not inverted.

29

Since the magnification m is $m = -i/p$, where p is the object distance and i is the image distance, $i = -mp$. Use this to substitute for i in $(1/p) + (1/i) = (1/f)$, where f is the focal length. The solve for p. The result is

$$p = f\left(1 - \frac{1}{m}\right) = (\pm 30\,\text{cm})\left(1 - \frac{1}{0.20}\right) = \pm 120\,\text{cm}.$$

Since p must be positive we must use the lower sign. Thus the focal length is -30 cm and the radius of curvature is $r = 2f == 60$ cm. Since the focal length and radius of curvature are negative the mirror is convex.

The object distance is 1.2 m and the image distance is $i = -mp = -(0.20)(120\,\text{cm}) = -24$ cm.

Since the image distance is negative the image is virtual and on the opposite side of the mirror from the object. Since the magnification is positive the image is not inverted.

35

Solve

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

for r. the result is

$$r = \frac{ip(n_2 - n_1)}{n_1 i + n_2 p} = \frac{(-13\,\text{cm})(+10\,\text{cm})}{((1.0)(-13\,\text{cm}) + (1.5)(+10\,\text{cm})} = -33\,\text{cm}.$$

Since the image distance is negative the image is virtual and appears on the same side of the surface as the object.

37

Solve

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

for r. the result is

$$i = \frac{n_2 r p}{(n_2 - n_1)p - n_1 r} = \frac{(1.0)(+30\,\text{cm})(+70\,\text{cm})}{(1.0 - 1.5)(+70\,\text{cm}) - (1.5)(+30\,\text{cm})} = -26\,\text{cm}.$$

Since the image distance is negative the image is virtual and appears on the same side of the surface as the object.

41

Use the lens maker's equation, Eq. 34–10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

Solve for r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\,\text{mm})}{2} = 45\,\text{mm}.$$

The radii are 45 mm and 90 mm.

47

The object distance p and image distance i obey $(1/p) + (1/i) = (1/f)$, where f is the focal length. In addition, $p+i = L$, where L (= 44 cm) is the distance from the slide to the screen. Use $i = L - p$ to substitute for i in the first equation and obtain $p^2 - pL + Lf = 0$. The solution is

$$p = \frac{L \pm \sqrt{L^2 - 4Lf}}{2} = \frac{(44\,\text{cm}) \pm \sqrt{4(44\,\text{cm})(11\,\text{cm})}}{2} = 22\,\text{cm}.$$

51

The lens is diverging, so the focal length is negative. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f} = \frac{(+8.0\,\text{cm})(-12\,\text{cm})}{(8.0\,\text{cm}) - (-12\,\text{cm})} = -4.8\,\text{cm}.$$

The magnification is $m = -i/p = -(-4.8\,\text{cm})/(+8.0\,\text{cm}) = 0.60$. Since the image distance is negative the image is virtual and appears on the same side of the lens as the object. Since the magnification is positive the image is not inverted.

55

The lens is converging, so the focal length is positive. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f} = \frac{(+45\,\text{cm})(+20\,\text{cm})}{(45\,\text{cm}) - (+20\,\text{cm})} = +36\,\text{cm}.$$

The magnification is $m = -i/p = -(36\,\text{cm})/(45\,\text{cm}) = -0.80$. Since the image distance is positive the image is real and appears on the opposite side of the lens from the object. Since the magnification is negative the image is inverted.

61

The focal length is

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = \frac{(+30\,\text{cm})(-42\,\text{cm})}{(1.55-1)[(-42\,\text{cm}) - (+30\,\text{cm})]} = +31.8\,\text{cm}\,.$$

Solve $(1/p) + (1/i) = (1/f)$ for i. the result is

$$i = \frac{pf}{p-f} = \frac{(+75\,\text{cm})(+31.8\,\text{cm})}{(+75\,\text{cm}) - (+31.8\,\text{cm})} = 55\,\text{cm}\,.$$

The magnification is $m = -i/p = -(55\,\text{cm})/(75\,\text{cm}) = -0.73$.

Since the image distance is positive the image is real and on the opposite side of the lends from the object. Since the magnification is negative the image is inverted.

75

Since $m = -i/p$, $i = -mp = -(+1.25)(+16\,\text{cm}) = -20\,\text{cm}$. Solve $(1/p) + (1/i) = (1/f)$ for f. The result is

$$f = \frac{pi}{p+i} = \frac{(+16\,\text{cm})(-20\,\text{cm})}{(+16\,\text{cm}) + (-20\,\text{cm})} = +80\,\text{cm}\,.$$

Since f is positive the lens is a converging lens. Since the image distance is negative the image is virtual and appears on the same side of the lens as the object. Since the magnification is positive the image is not inverted.

79

The image is on the same side of the lens as the object. This means that the image is virtual and the image distance is negative. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f}\,.$$

and the magnification is

$$m = -\frac{i}{p} = -\frac{f}{p-f}\,.$$

Since the magnification is less than 1.0, f must be negative and the lens must be a diverging lens. The image distance is

$$i = \frac{(+5.0\,\text{cm})(-10\,\text{cm})}{(5.0\,\text{cm}) - (-10\,\text{cm})} = -3.3\,\text{cm}\,.$$

and the magnification is $m = -i/p = -(-3.3\,\text{cm})/(5.0\,\text{cm}) = 0.66\,\text{cm}$.

Since the magnification is positive the image is not inverted.

81

Lens 1 is converging and so has a positive focal length. Solve $(1/p_1) + (1/i_1) = (1/f_1)$ for the image distance i_1 associated with the image produced by this lens. The result is

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20\,\text{cm})(+9.0\,\text{cm})}{(20\,\text{cm}) - (9.0\,\text{cm})} = 16.4\,\text{cm}\,.$$

This image is the object for lens 2. The object distance is $d - p_2 = (8.0\,\text{cm}) - (16.4\,\text{cm}) = -8.4\,\text{cm}$. The negative sign indicates that the image is behind the second lens. The lens equation is still valid. The second lens has a positive focal length and the image distance for the image it forms is

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4\,\text{cm})(5.0\,\text{cm})}{(-8.4\,\text{cm}) - (5.0\,\text{cm})} = +3.1\,\text{cm}\,.$$

The overall magnification is the product of the individual magnifications:

$$m = m_1 m_2 = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right) = \left(-\frac{16.4\,\text{cm}}{20\,\text{cm}}\right)\left(-\frac{3.1\,\text{cm}}{-8.4\,\text{cm}}\right) = -0.30\,.$$

Since the final image distance is positive the final image is real and on the opposite side of lens 2 from the object. Since the magnification is negative the image is inverted.

89

(a) If L is the distance between the lenses, then according to Fig. 34–20, the tube length is $s = L - f_{\text{ob}} - f_{\text{ey}} = 25.0\,\text{cm} - 4.00\,\text{cm} - 8.00\,\text{cm} = 13.0\,\text{cm}$.

(b) Solve $(1/p) + (1/i) = (1/f_{\text{ob}})$ for p. The image distance is $i = f_{\text{ob}} + s = 4.00\,\text{cm} + 13.0\,\text{cm} = 17.0\,\text{cm}$, so

$$p = \frac{i f_{\text{ob}}}{i - f_{\text{ob}}} = \frac{(17.0\,\text{cm})(4.00\,\text{cm})}{17.0\,\text{cm} - 4.00\,\text{cm}} = 5.23\,\text{cm}\,.$$

(c) The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{17.0\,\text{cm}}{5.23\,\text{cm}} = -3.25\,.$$

(d) The angular magnification of the eyepiece is

$$m_\theta = \frac{25\,\text{cm}}{f_{\text{ey}}} = \frac{25\,\text{cm}}{8.00\,\text{cm}} = 3.13\,.$$

(e) The overall magnification of the microscope is

$$M = m m_\theta = (-3.25)(3.13) = -10.2\,.$$

93

(a) When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. Set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus $i = f = 2.50\,\text{cm}$. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change. If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'} .$$

Substitute $i = f$ and solve for f'. You should obtain

$$f' = \frac{pf}{f+p} = \frac{(40.0\,\text{cm})(2.50\,\text{cm})}{40.0\,\text{cm} + 2.50\,\text{cm}} = 2.35\,\text{cm} .$$

(b) Consider the lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) ,$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34–46, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of either or both radii.

103

For a thin lens, $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. Solve for i:

$$i = \frac{fp}{p-f} .$$

Let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. Then

$$i = \frac{f(f+x)}{x} .$$

Now let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside. Then

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

and $xx' = f^2$.

105

Place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$.

The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thicknesses of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive. In the thin lens equation, replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}\,.$$

The solution for f is

$$f = \frac{f_1 f_2}{f_1 + f_2}\,.$$

<u>107</u>

(a) and (b) Since the height of the image is twice the height of the fly and since the fly and its image have the same orientation the magnification of the lens is $m = +2.0$. Since $m = -i/p$, where p is the object distance and i is the image distance, $i = -2p$. Now $|p+i| = d$, so $|-p| = d$ and $p = d = 20\,\text{cm}$. The image distance is $-40\,\text{cm}$.

Solve $(1/p) + (1/i) = (1/f)$ for f. the result is

$$f = \frac{pi}{p+i} = \frac{(20\,\text{cm})(-40\,\text{cm})}{(20\,\text{cm}) + (-40\,\text{cm})} = +40\,\text{cm}\,.$$

(c) and (d) Now $m = +0.5$ and $i = -0.5p$. Since $|p+i| = d$, $0.5p = d$ and $p = 2d = 40\,\text{cm}$. The image distance is $-20\,\text{cm}$ and the focal length is

$$f = \frac{pi}{p+i} = \frac{(40\,\text{cm})(-20\,\text{cm})}{(40\,\text{cm}) + (-20\,\text{cm})} = -40\,\text{cm}\,.$$

Chapter 35

5

(a) Take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where k_1 ($= 2\pi/\lambda_1$) is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2L - \omega t$, where k_2 ($= 2\pi/\lambda_2$) is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L\,.$$

Now $\lambda_1 = \lambda_{\text{air}}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{\text{air}}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is $\phi_1 - \phi_2 = (2\pi/\lambda_{\text{air}})(n_1 - n_2)L$. The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{\text{air}}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\,\text{m}\,.$$

(b) 5.65 rad is less than 2π rad ($= 6.28$ rad), the phase difference for completely constructive interference, and greater than π rad ($= 3.14$ rad), the phase difference for completely destructive interference. The interference is therefore intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

15

Interference maxima occur at angles θ such that $d\sin\theta = m\lambda$, where d is the separation of the sources, λ is the wavelength, and m is an integer. Since $d = 2.0$ m and $\lambda = 0.50$ m, this means that $\sin\theta = 0.25m$. You want all values of m (positive and negative) for which $|0.25m| \leq 1$. These are -4, -3, -2, -1, 0, +1, +2, +3, and +4. For each of these except -4 and +4, there are two different values for θ. A single value of θ ($-90°$) is associated with $m = -4$ and a single value ($-90°$) is associated with $m = +4$. There are sixteen different angles in all and therefore sixteen maxima.

17

The angular positions of the maxima of a two-slit interference pattern are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then $d\theta = m\lambda$. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. Let λ' be the wavelength for which the angular separation is 10.0% greater. Then $1.10\lambda/d = \lambda'/d$ or $\lambda' = 1.10\lambda = 1.10(589\,\text{nm}) = 648\,\text{nm}$.

19

The condition for a maximum in the two-slit interference pattern is $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then $d\theta = m\lambda$ and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m+1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by $\Delta y = D\,\Delta\theta = \lambda D/d$. Thus

$$\Delta y = \frac{(500 \times 10^{-9}\,\text{m})(5.40\,\text{m})}{1.20 \times 10^{-3}\,\text{m}} = 2.25 \times 10^{-3}\,\text{m} = 2.25\,\text{mm}\,.$$

21

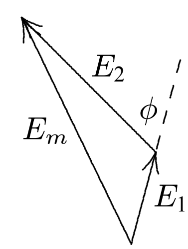
The maxima of a two-slit interference pattern are at angles θ that are given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$ and the separation on a screen a distance D away is

$$\begin{aligned}\Delta y &= D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)\\ &= \left[\frac{3(1.0\,\text{m})}{5.0 \times 10^{-3}\,\text{m}}\right](600 \times 10^{-9}\,\text{m} - 480 \times 10^{-9}\,\text{m}) = 7.2 \times 10^{-5}\,\text{m}\,.\end{aligned}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ was made. $\Delta\theta$ must be in radians.

29

The phasor diagram is shown to the right. Here $E_1 = 1.00$, $E_2 = 2.00$, and $\phi = 60°$. The resultant amplitude E_m is given by the trigonometric law of cosines:



$$E_m^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(180° - \phi)\,,$$

so

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65\,.$$

39

For complete destructive interference, you want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. Solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for L. Here m is an integer. The result is

$$L = \frac{(2m+1)\lambda}{4n}\,.$$

To find the least thickness for which destructive interference occurs, take $m = 0$. Then

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9}\,\text{m}}{4(1.25)} = 1.2 \times 10^{-7}\,\text{m}\,.$$

<u>**41**</u>

Since n_1 is greater than n_2 there is no change in phase on reflection from the first surface. Since n_2 is less than n_3 there is a change in phase of π rad on reflection from the second surface. One wave travels a distance $2L$ further than the other, so the difference in the phases of the two waves is $4\pi L/\lambda_2 + \pi$, where λ_2 is the wavelength in medium 2. Since interference produces a minimum the phase difference must be an odd multiple of π. Thus $4\pi L/\lambda_2 + \pi = (2m+1)\pi$, where m is an integer or zero. Replace λ_2 with λ/n_2, where λ is the wavelength in air, and solve for λ. The result is

$$\lambda = \frac{4Ln_2}{2m} = \frac{2(380\,\text{nm})(1.1.34)}{m} = \frac{1018\,\text{nm}}{m}\,.$$

For $m = 1$, $\lambda = 1018\,\text{nm}$ and for $m = 2$, $\lambda = (1018\,\text{nm})/2 = 509\,\text{nm}$. Other wavelengths are shorter. Only $\lambda = 509\,\text{nm}$ is in the visible range.

<u>**47**</u>

There is a phase shift on reflection of π for both waves and one wave travels a distance $2L$ further than the other, so the phase difference of the reflected waves is $4\pi L/\lambda_2$, where λ_2 is the wavelength in medium 2. Since the result of the interference is a minimum of intensity the phase difference must be an odd multiple of π. Thus $4\pi L/\lambda_2 = (2m+1)\pi$, where m is an integer or zero. Replace λ_2 with λ/n_2, where λ is the wavelength in air, and solve for λ. The result is

$$\lambda = \frac{4Ln_2}{2m+1} = \frac{4(210\,\text{nm})(1.46)}{2m+1} = \frac{1226\,\text{nm}}{2m+1}\,.$$

For $m = 1$, $\lambda = (1226\,\text{nm})/3 = 409\,\text{nm}$. This is in the visible range. Other values of m are associated with wavelengths that are not in the visible range.

<u>**53**</u>

(a) Oil has a greater index of refraction than air and water has a still greater index of refraction. There is a change of phase of π rad at each reflection. One wave travels a distance $2L$ further than the other, where L is the thickness of the oil. The phase difference of the two reflected waves is $4\pi L/\lambda_o$, where λ is the wavelength in oil, and this must be equal to a multiple of 2π for a bright reflection. Thus $4\pi L/\lambda_o = 2m\pi$, where m is an integer. Use $\lambda = n_o\lambda_o$, where n_o is the index of refraction for oil, to find the wavelength in air. The result is

$$\lambda = \frac{2n_oL}{m} = \frac{2(1.20)(460\,\mathrm{nm})}{m} = \frac{1104\,\mathrm{nm}}{m}\,.$$

For $m = 1$, $\lambda = 1104\,\mathrm{nm}$; for $m = 2$, $\lambda = (1104\,\mathrm{nm})/2 = 552\,\mathrm{nm}$; and for $m = 3$, $\lambda = (1104\,\mathrm{nm})/3 = 368\,\mathrm{nm}$. Other wavelengths are shorter. Only $\lambda = 552\,\mathrm{nm}$ is in the visible range.

(b) A maximum in transmission occurs for wavelengths for which the reflection is a minimum. The phases of the two reflected waves then differ by an odd multiple of π rad. This means $4\pi L/\lambda_o = (2m + 1)\pi$ and

$$\lambda = \frac{4n_oL}{2m+1} = \frac{4(1.20)(460\,\mathrm{nm})}{2m+1} = \frac{2208\,\mathrm{nm}}{2m+1}\,.$$

For $m = 0$, $\lambda = 2208\,\mathrm{nm}$; for $m = 1$, $\lambda = (2208\,\mathrm{nm})/3 = 736\,\mathrm{nm}$; and for $m = 3$, $\lambda = (2208\,\mathrm{nm})/5 = 442\,\mathrm{nm}$. Other wavelengths are shorter. Only $\lambda = 442\,\mathrm{nm}$ falls in the visible range.

63

One wave travels a distance $2L$ further than the other. This wave is reflected twice, once from the back surface and once from the front surface. Since n_2 is greater than n_3 there is no change in phase at the back-surface reflection. Since n_1 is greater than n_2 there is a phase change of π at the front-surface reflection. Thus the phase difference of the two waves as they exit material 2 is $4\pi L/\lambda_2 + \pi$, where λ_2 is the wavelength in material 2. For a maximum in intensity the phase difference is a multiple of 2π. Thus $4\pi L/\lambda_2 + \pi = 2m\pi$, where m is an integer. The solution for λ_2 is

$$\lambda_2 = \frac{4L}{2m-1} = \frac{4(415\,\mathrm{nm})}{2m-1} = \frac{1660\,\mathrm{nm}}{2m-1}\,.$$

The wavelength in air is

$$\lambda = n_2\lambda_2 = \frac{(1.59)(1660\,\mathrm{nm})}{2m-1} = \frac{2639\,\mathrm{nm}}{2m-1}\,.$$

For $m = 1$, $\lambda = 2639\,\mathrm{nm}$; for $m = 2$, $\lambda = 880\,\mathrm{nm}$; for $m = 3$, $\lambda = 528\,\mathrm{nm}$; and for $m = 4$, $\lambda = 377\,\mathrm{nm}$. Other wavelengths are shorter. Only $\lambda = 528\,\mathrm{nm}$ is in the visible range.

71

Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is L, the condition for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$, where λ (= 683 nm) is the wavelength and m is an integer. This is satisfied for $m = 140$:

$$L = \frac{(m+\frac{1}{2})\lambda}{2} = \frac{(140.5)(683 \times 10^{-9}\,\mathrm{m})}{2} = 4.80 \times 10^{-5}\,\mathrm{m} = 0.048\,\mathrm{mm}\,.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

75

Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π-rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda$, where λ is the wavelength in air and m is an integer. Thus $d = (2m + 1)\lambda/4$. As the geometry of Fig. 35–47 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus $(2m + 1)\lambda/4 = R - \sqrt{R^2 - r^2}$. Solve for r. First rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m + 1)\lambda}{4} .$$

Now square both sides and solve for r^2. When you take the square root, you should get

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2} - \frac{(2m + 1)^2\lambda^2}{16}} .$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2}} .$$

81

Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it and let ϕ_2 be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n - 1)L}{\lambda} ,$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror.

Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated,

$$\frac{4\pi(n - 1)L}{\lambda} = 2N\pi$$

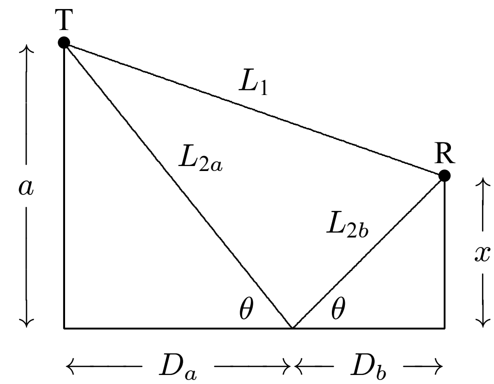
and

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500 \times 10^{-9}\,\text{m})}{2(5.0 \times 10^{-2}\,\text{m})} = 1.00030 .$$

87

Suppose the wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2. Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver the difference $L_2 - L_2$ in the distances traveled must be an odd multiple of a half wavelength.

Look at the diagram on the right. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$ and the right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line gives $D_b = x/\tan\theta$. Since $D_a + D_b = D$,



$$\tan\theta = \frac{a+x}{D}\,.$$

Use the identity $\sin^2\theta = \tan^2\theta/(1+\tan^2\theta)$ to show that $\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2}$. This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2+(a+x)^2}}{a+x}\,,$$

so

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2+(a+x)^2}}{a+x} = \sqrt{D^2+(a+x)^2}\,.$$

Use the binomial theorem, with D^2 large and $a^2 + x^2$ small, to approximate this expression: $L_2 \approx D + (a+x)^2/2D$.

The distance traveled by the direct wave is $L_1 = \sqrt{D^2+(a-x)^2}$. Use the binomial theorem to approximate this expression: $L_1 \approx D + (a-x)^2/2D$. Thus

$$L_2 - L_1 \approx D + \frac{a^2+2ax+x^2}{2D} - D - \frac{a^2-2ax+x^2}{2D} = \frac{2ax}{D}\,.$$

Set this equal to $(m+\frac{1}{2})\lambda$, where m is zero or a positive integer. Solve for x. The result is $x = (m+\frac{1}{2})(D/2a)\lambda$.

89

Bright fringes occur at an angle θ such that $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength in the medium of propagation, and m is an integer. Near the center of the pattern the angles are small and $\sin\theta$ can be approximated by θ in radians. Thus $\theta = m\lambda/d$ and the angular separation of two adjacent bright fringes is $\Delta\theta = \lambda/d$. When the arrangement is immersed in water the angular separation of the fringes becomes $\Delta\theta' = \lambda_w/d$, where λ_w is the wavelength in water. Since $\lambda_w = \lambda/n_w$, where n_w is the index of refraction of water, $\Delta\theta' = \lambda/n_w d = (\Delta\theta)/n_w$. Since the units of the angles cancel from this equation we may substitute the angles in degrees and obtain $\Delta\theta' = 0.30°/1.33 = 0.23°$.

93

(a) For wavelength λ dark bands occur where the path difference is an odd multiple of $\lambda/2$. That is, where the path difference is $(2m+1)\lambda/2$, where m is an integer. The fourth dark band from the central bright fringe is associated with $m = 3$ and is $7\lambda/2 = 7(500\,\text{nm})/2 = 1750\,\text{nm}$.

(b) The angular position θ of the first bright band on either side of the central band is given by $\sin\theta = \lambda/d$, where d is the slit separation. The distance on the screen is given by $\Delta y = D\tan\theta$, where D is the distance from the slits to the screen. Because θ is small its sine and tangent are very nearly equal and $\Delta y = D\sin\theta = D\lambda/d$.

Dark bands have angular positions that are given by $\sin\theta = (m+\frac{1}{2})\lambda/d$ and, for the fourth dark band, $m = 3$ and $\sin\theta_4 = (7/2)\lambda/d$. Its distance on the screen from the central fringe is $\Delta y_4 = D\tan\theta_4 = D\sin\theta_4 = 7D\lambda/2d$. This means that $D\lambda/d = 2\Delta y_4/7 = 2(1.68\,\text{cm})/7 = 0.48\,\text{cm}$. Note that this is Δy.

99

Minima occur at angles θ for which $\sin\theta = (m+\frac{1}{2})\lambda/d$, where λ is the wavelength, d is the slit separation, and m is an integer. For the first minimum, $m = 0$ and $\sin\theta_1 = \lambda/2d$. For the tenth minimum, $m = 9$ and $\sin\theta_{10} = 19\lambda/2d$.

The distance on the screen from the central fringe to a minimum is $y = D\tan\theta$, where D is the distance from the slits to the screen. Since the angle is small we may approximate its tangent with its sine and write $y = D\sin\theta = D(m+\frac{1}{2})\lambda/d$. Thus the separation of the first and tenth minima is

$$\Delta y = \frac{D}{d}\left(\frac{19\lambda}{2} - \frac{\lambda}{2}\right) = \frac{9D\lambda}{d}$$

and

$$\lambda = \frac{d\,\Delta y}{9D} = \frac{(0.150\times10^{-3}\,\text{m})(18.0\times10^{-3}\,\text{m})}{9(50.0\times10^{-2}\,\text{m})} = 6.00\times10^{-7}\,\text{m}\,.$$

103

The difference in the path lengths of the two beams is $2x$, so their difference in phase when they reach the detector is $\phi = 4\pi x/\lambda$, where λ is the wavelength. Assume their amplitudes are the same. According to Eq. 35–22 the intensity associated with the addition of two waves is proportional to the square of the cosine of half their phase difference. Thus the intensity of the light observed in the interferometer is proportional to $\cos^2(2\pi x/\lambda)$. Since the intensity is maximum when $x = 0$ (and the arms have equal lengths), the constant of proportionality is the maximum intensity I_m and $I = I_m\cos^2(2\pi x/\lambda)$.

Chapter 36

9

The condition for a minimum of intensity in a single-slit diffraction pattern is $a \sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. To find the angular position of the first minimum to one side of the central maximum, set $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{589 \times 10^{-9}\,\mathrm{m}}{1.00 \times 10^{-3}\,\mathrm{m}}\right) = 5.89 \times 10^{-4}\,\mathrm{rad}\,.$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is $y_1 = D\tan\theta_1 = (3.00\,\mathrm{m})\tan(5.89 \times 10^{-4}\,\mathrm{rad}) = 1.767 \times 10^{-3}\,\mathrm{m}$.

To find the second minimum, set $m = 2$:

$$\theta_2 = \sin^{-1}\left[\frac{2(589 \times 10^{-9}\,\mathrm{m})}{1.00 \times 10^{-3}\,\mathrm{m}}\right] = 1.178 \times 10^{-3}\,\mathrm{rad}\,.$$

The distance from the pattern center to the minimum is $y_2 = D\tan\theta_2 = (3.00\,\mathrm{m})\tan(1.178 \times 10^{-3}\,\mathrm{rad}) = 3.534 \times 10^{-3}\,\mathrm{m}$. The separation of the two minima is $\Delta y = y_2 - y_1 = 3.534\,\mathrm{mm} - 1.767\,\mathrm{mm} = 1.77\,\mathrm{mm}$.

17

(a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}\,,$$

where $\alpha = (\pi a/\lambda)\sin\theta$, a is the slit width and λ is the wavelength. The angle θ is measured from the forward direction. You want $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2\,.$$

(b) Evaluate $\sin^2\alpha$ and $\alpha^2/2$ for $\alpha = 1.39\,\mathrm{rad}$ and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, you should also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda)\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right)\,.$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right)\,.$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5^\circ.$$

(e) For $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1^\circ.$$

(f) For $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06^\circ.$$

21

(a) Use the Rayleigh criteria. To resolve two point sources, the central maximum of the diffraction pattern of one must lie at or beyond the first minimum of the diffraction pattern of the other. This means the angular separation of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. For the headlights of this problem,

$$\theta_R = \frac{1.22(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.3 \times 10^{-4}\,\text{rad}.$$

(b) If D is the distance from the headlights to the eye when the headlights are just resolvable and ℓ is the separation of the headlights, then $\ell = D\tan\theta_R \approx D\theta_R$, where the small angle approximation $\tan\theta_R \approx \theta_R$ was made. This is valid if θ_R is measured in radians. Thus $D = \ell/\theta_R = (1.4\,\text{m})/(1.34 \times 10^{-4}\,\text{rad}) = 1.0 \times 10^4\,\text{m} = 10\,\text{km}$.

25

(a) Use the Rayleigh criteria: two objects can be resolved if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$, where λ is the wavelength of the light and d is the diameter of the aperture (the eye or mirror). If D is the distance from the observer to the objects, then the smallest separation ℓ they can have and still be resolvable is $\ell = D\tan\theta_R \approx D\theta_R$, where θ_R is measured in radians. The small angle approximation $\tan\theta_R \approx \theta_R$ was made. Thus

$$\ell = \frac{1.22D\lambda}{d} = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.1 \times 10^{7}\,\text{m} = 1.1 \times 10^{4}\,\text{km}.$$

This distance is greater than the diameter of Mars. One part of the planet's surface cannot be resolved from another part.

(b) Now $d = 5.1$ m and

$$\ell = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 1.1 \times 10^{4}\,\text{m} = 11\,\text{km}.$$

29

(a) The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{220 \times 10^9\,\mathrm{Hz}} = 1.36 \times 10^{-3}\,\mathrm{m}\,.$$

Thus

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(1.36 \times 10^{-3}\,\mathrm{m})}{55.0 \times 10^{-2}\,\mathrm{m}}\right) = 3.02 \times 10^{-3}\,\mathrm{rad}\,.$$

The angular width of the central maximum is twice this, or 6.04×10^{-3} rad ($0.346°$).

(b) Now $\lambda = 1.6$ cm and $d = 2.3$ m, so

$$\theta = \sin^{-1}\left(\frac{1.22(1.6 \times 10^{-2}\,\mathrm{m})}{2.3\,\mathrm{m}}\right) = 8.5 \times 10^{-3}\,\mathrm{rad}\,.$$

The angular width of the central maximum is 1.7×10^{-2} rad ($0.97°$).

39

(a) The angular positions θ of the bright interference fringes are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so you want to count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or what is the same, the number of values of m for which $-\sin\theta_1 < \sin\theta < +\sin\theta_1$. This means $-1/a < m/d < 1/a$ or $-d/a < m < +d/a$. Now $d/a = (0.150 \times 10^{-3}\,\mathrm{m})/(30.0 \times 10^{-6}\,\mathrm{m}) = 5.00$, so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are nine fringes.

(b) The intensity at the screen is given by

$$I = I_m\left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2,$$

where $\alpha = (\pi a/\lambda)\sin\theta$, $\beta = (\pi d/\lambda)\sin\theta$, and I_m is the intensity at the center of the pattern. For the third bright interference fringe, $d\sin\theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2\beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi}\right)^2 = 0.255\,.$$

The intensity ratio is $I/I_m = 0.255$.

45

The ruling separation is $d = 1/(400\,\mathrm{mm}^{-1}) = 2.5 \times 10^{-3}$ mm. Diffraction lines occur at angles θ such that $d\sin\theta = m\lambda$, where λ is the wavelength and m is an integer. Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. Take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$. Since $d/\lambda = (2.5\times10^{-6}\,\text{m})/(700\times10^{-9}\,\text{m}) = 3.57$, that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

51

(a) Maxima of a diffraction grating pattern occur at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The two lines are adjacent, so their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m+1$ be the order number for the line with $\sin\theta = 0.3$. Then $0.2d = m\lambda$ and $0.3d = (m+1)\lambda$. Subtract the first equation from the second to obtain $0.1d = \lambda$, or $d = \lambda/0.1 = (600\times10^{-9}\,\text{m})/0.1 = 6.0\times10^{-6}\,\text{m}$.

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a\sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a\sin\theta = \lambda$. It is also given by $d\sin\theta = 4\lambda$, so $a = d/4 = (6.0\times10^{-6}\,\text{m})/4 = 1.5\times10^{-6}\,\text{m}$.

(c) First, set $\theta = 90°$ and find the largest value of m for which $m\lambda < d\sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since $d/\lambda = (6.0\times10^{-6}\,\text{m})/(600\times10^{-9}\,\text{m}) = 10.0$, the highest order seen is the $m = 9$ order.

(d) and (e) The fourth and eighth orders are missing so the observable orders are $m = 0$, 1, 2, 3, 5, 6, 7, and 9. The second highest order is the $m = 7$ order and the third highest order is the $m = 6$ order.

61

If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{\text{avg}}/\Delta\lambda$. The text shows this is Nm, where N is the number of rulings in the grating and m is the order of the lines. Thus $\lambda_{\text{avg}}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{\text{avg}}}{m\,\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3.65\times10^3\ \text{rulings}\,.$$

73

We want the reflections to obey the Bragg condition $2d\sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. Solve for θ:

$$\theta = \sin^{-1}\left[\frac{m\lambda}{2d}\right] = \sin^{-1}\left[\frac{(0.125\times10^{-9}\,\text{m})m}{2(0.252\times10^{-9}\,\text{m})}\right] = \sin^{-1}(0.2480m)\,.$$

For $m = 1$ this gives $\theta = 14.4°$. The crystal should be turned $45° - 14.4° = 30.6°$ clockwise.

For $m = 2$ it gives $\theta = 29.7°$. The crystal should be turned $45° - 29.7° = 15.3°$ clockwise.

For $m = 3$ it gives $\theta = 48.1°$. The crystal should be turned $48.1° - 45° = 3.1°$ counterclockwise.

For $m = 4$ it gives $\theta = 82.8°$. The crystal should be turned $82.8° - 45° = 37.8°$ counterclockwise. There are no intensity maxima for $m > 4$ as you can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4. For clockwise turns the smaller value is 15.3° and the larger value is 30.6°. For counterclockwise turns the smaller value is 3.1° and the larger value is 37.8°.

77

Intensity maxima occur at angles θ such that $d \sin\theta = m\lambda$, where d is the separation of adjacent rulings and λ is the wavelength. Here the ruling separation is $1/(200\,\text{mm}^{-1}) = 5.00 \times 10^{-3}\,\text{mm} = 5.00 \times 10^{-6}\,\text{m}$. Thus

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5.00 \times 10^{-6}\,\text{m})\sin 30.0°}{m} = \frac{2.50 \times 10^{-6}\,\text{m}}{m}\,.$$

For $m = 1$, $\lambda = 2500\,\text{nm}$; for $m = 2$, $\lambda = 1250\,\text{nm}$; for $m - 3$, $\lambda = 833\,\text{nm}$; for $m = 4$, $\lambda = 625\,\text{nm}$; for $m = 5$, $\lambda = 500\,\text{nm}$, and for $m = 6$, $\lambda = 417\,\text{nm}$. Only the last three are in the visible range, so the longest wavelength in the visible range is 625 nm, the next longest is 500 nm, and the third longest is 417 nm.

79

Suppose m_o is the order of the minimum for orange light, with wavelength λ_o, and m_{bg} is the order of the minimum for blue-green light, with wavelength λ_{bg}. Then $a\sin\theta = m_o\lambda_o$ and $a\sin\theta = m_{bg}\lambda_{bg}$. Thus $m_o\lambda_o = m_{bg}\lambda_{bg}$ and $m_{bg}/m_o = \lambda_o/\lambda_{bg} = (600\,\text{nm})/(500\,\text{nm}) = 6/5$. The smallest two integers with this ratio are $m_{bg} = 6$ and $m_o = 5$. The slit width is

$$a = \frac{m_o\lambda_o}{\sin\theta} = \frac{5(600 \times 10^{-9}\,\text{m}}{\sin(1.00 \times 10^{-3}\,\text{rad})} = 3.0 \times 10^{-3}\,\text{m}\,.$$

Other values for m_o and m_{bg} are possible but these are associated with a wider slit.

81

(a) Since the first minimum of the diffraction pattern occurs at the angle θ such that $\sin\theta = \lambda/a$, where λ is the wavelength and a is the slit width, the central maximum extends from $\theta_1 = -\sin^{-1}(\lambda/a)$ to $\theta_2 = +\sin^{-1}(\lambda/a)$. Maxima of the two-slit interference pattern are at angles θ such that $\sin\theta = m\lambda/d$, where d is the slit separation and m is an integer. We wish to know the number of values of m such that $\sin^{-1}(m\lambda/d)$ lies between $-\sin^{-1}(\lambda/a)$ and $+\sin(\lambda/a)$ or, what is the same, the number of values of m such that m/d lies between $-1/a$ and $+1/a$. The greatest m can be is the greatest integer that is smaller than $d/a = (14\,\mu\text{m})/(2.0\,\mu\text{m}) = 7$. (The $m = 7$ maximum does not appear since it coincides with a minimum of the diffraction pattern.) There are 13 such values: 0, ± 1, ± 2, ± 3; ± 4; ± 5, and ± 6. Thus 13 interference maxima appear in the central diffraction envelope.

(b) The first diffraction envelope extends from $\theta_1 = \sin^{-1}(\lambda/a)$ to $\theta_2 = \sin^{-1}(2\lambda/a)$. Thus we wish to know the number of values of m such that m/d is greater than $1/a$ and less than $2/a$. Since $d = 7.0a$, m can be 8, 9, 10, 11, 12, or 13. That is, there are 6 interference maxima in the first diffraction envelope.

93

If you divide the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor, then at the central maximum in the diffraction pattern you add N phasors, all in the same direction and each with the same amplitude. The intensity there is proportional to N^2. If you double the slit width, you need $2N$ phasors if they are each to have the amplitude of the phasors you used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

95

(a) Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\,\Delta\lambda + \lambda\,\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\,\Delta f = -\frac{\lambda^2}{c}\,\Delta f\,,$$

where $f = c/\lambda$ was used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength. We may interpret Δf as the range of frequencies that can be resolved and take it to be positive. Then

$$\frac{\lambda^2}{c}\,\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}\,.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d\sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}\,.$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substitute the expressions you derived for Δt and Δf to obtain

$$\Delta f\,\Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{Nd\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1\,.$$

The condition $d\sin\theta = m\lambda$ for a diffraction line was used to obtain the last result.

101

The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\ell \sin\theta = m\lambda$, where ℓ is the slit separation and m is an integer. Differentiate this with respect to θ to obtain $(d\theta/d\lambda)\,\ell\cos\theta = m$ or

$$D = \frac{\ell\theta}{\ell\lambda} = \frac{m}{\ell\cos\theta}\,.$$

Now $m = (\ell/\lambda)\sin\theta$, so

$$D = \frac{\ell\sin\theta}{\ell\lambda\cos\theta} = \frac{\tan\theta}{\lambda}\,.$$

The trigonometric identity $\tan\theta = \sin\theta/\cos\theta$ was used.

Chapter 37

11

(a) The rest length L_0 (= 130 m) of the spaceship and its length L as measured by the timing station are related by $L = L_0/\gamma = L_0\sqrt{1-\beta^2}$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Thus $L = (130\,\mathrm{m})\sqrt{1-(0.740)^2} = 87.4\,\mathrm{m}$.

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\mathrm{m}}{(0.740)(2.9979\times10^8\,\mathrm{m/s})} = 3.94\times10^{-7}\,\mathrm{s}\,.$$

19

The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma[x - vt]$$
$$t' = \gamma[t - \beta x/c]\,,$$

where $\beta = v/c = 0.950$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.2026$. Thus

$$\begin{aligned} x' &= (3.2026)\left[100\times10^3\,\mathrm{m} - (0.950)(2.9979\times10^8\,\mathrm{m/s})(200\times10^{-6}\,\mathrm{s}\right] \\ &= 1.38\times10^5\,\mathrm{m} = 138\,\mathrm{km} \end{aligned}$$

and

$$t' = (3.2026)\left[200\times10^{-6}\,\mathrm{s} - \frac{(0.950)(100\times10^3\,\mathrm{m})}{2.9979\times10^8\,\mathrm{m/s}}\right] = -3.74\times10^{-4}\,\mathrm{s} = -374\,\mu\mathrm{s}\,.$$

21

(a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25\,.$$

(b) In the unprimed frame, the time for the clock to travel from the origin to $x = 180\,\mathrm{m}$ is

$$t = \frac{x}{v} = \frac{180\,\mathrm{m}}{(0.600)(2.9979\times10^8\,\mathrm{m/s})} = 1.00\times10^{-6}\,\mathrm{s}\,.$$

The proper time interval between the two events (at the origin and at $x = 180\,\mathrm{m}$) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00\times10^{-6}\,\mathrm{s}}{1.25} = 8.00\times10^{-7}\,\mathrm{s}\,.$$

29

Use Eq. 37–29 with $u' = 0.40c$ and $v = 0.60c$. Then

$$u = \frac{0.40c + 0.60c}{1 + (0.40c)(0.60c)/c^2} = 0.81c\,.$$

33

Calculate the speed of the micrometeorite relative to the spaceship. Let S' be the reference frame for which the data is given and attach frame S to the spaceship. Suppose the micrometeorite is going in the positive x direction and the spaceship is going in the negative x direction, both as viewed from S'. Then, in Eq. 37–29, $u' = 0.82c$ and $v = 0.82c$. Notice that v in the equation is the velocity of S' relative to S. Thus the velocity of the micrometeorite in the frame of the spaceship is

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.82c + 0.82c}{1 + (0.82c)(0.82c)/c^2} = 0.9806c\,.$$

The time for the micrometeorite to pass the spaceship is

$$\Delta t = \frac{L}{u} = \frac{350\,\text{m}}{(0.9806)(2.9979 \times 10^8\,\text{m/s})} = 1.19 \times 10^{-6}\,\text{s}\,.$$

37

The spaceship is moving away from Earth, so the frequency received is given by

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}\,,$$

where f_0 is the frequency in the frame of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to Earth. See Eq. 37–31. Thus

$$f = (100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9\,\text{MHz}\,.$$

39

The spaceship is moving away from Earth, so the frequency received is given by

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}\,,$$

where f_0 is the frequency in the frame of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to Earth. See Eq. 37–31. The frequency f and wavelength λ are related by $f\lambda = c$, so if λ_0 is the wavelength of the light as seen on the spaceship and λ is the wavelength detected on Earth, then

$$\lambda = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}} = (450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}} = 550\,\text{nm}\,.$$

This is in the yellow-green portion of the visible spectrum.

<u>**43**</u>

Use the two expressions for the total energy: $E = mc^2 + K$ and $E = \gamma mc^2$, where m is the mass of an electron, K is the kinetic energy, and $\gamma = 1/\sqrt{1-\beta^2}$. Thus $mc^2 + K = \gamma mc^2$ and

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{(100.000 \times 10^6\,\text{eV})(1.602\,176\,462\,\text{J/eV})}{(9.109\,381\,88 \times 10^{-31}\,\text{kg})(2.997\,924\,58 \times 10^8\,\text{m/s})^2} = 196.695\,.$$

Now $\gamma^2 = 1/(1-\beta^2)$, so

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(196.695)^2}} = 0.999\,987\,.$$

<u>**53**</u>

The energy equivalent of one tablet is $mc^2 = (320 \times 10^{-6}\,\text{kg})(2.9979 \times 10^8\,\text{m/s})^2 = 2.88 \times 10^{13}\,\text{J}$. This provides the same energy as $(2.88 \times 10^{13}\,\text{J})/(3.65 \times 10^7\,\text{J/L}) = 7.89 \times 10^5\,\text{L}$ of gasoline. The distance the car can go is $d = (7.89 \times 10^5\,\text{L})(12.75\,\text{km/L}) = 1.01 \times 10^7\,\text{km}$.

<u>**71**</u>

The energy of the electron is given by $E = mc^2/\sqrt{1-(v/c)^2}$, which yields

$$v = \sqrt{1 - \left[\frac{mc^2}{E}\right]^2}\,c = \sqrt{1 - \left[\frac{(9.11 \times 10^{-31}\,\text{kg})(2.9979 \times 10^8\,\text{m/s})^2}{(1533\,\text{MeV})(1.602 \times 10{-}13\,\text{J/MeV})}\right]^2} = 0.99999994c \approx c$$

for the speed v of the electron. In the rest frame of Earth the trip took time $t = 26\,\text{y}$. A clock traveling with the electron records the proper time of the trip, so the trip in the rest frame of the electron took time $t' = t/\gamma$. Now

$$\gamma = \frac{E}{mc^2} = \frac{1533\,\text{MeV})(1.602 \times 10^{-13}\,\text{J/MeV})}{(9.11 \times 10^{-31}\,\text{kg})(2.9979 \times 10^8\,\text{m/s})} = 3.0 \times 10^3$$

and $t' = (26\,\text{y})/(3.0 \times 10^3) = 8.7 \times 10^{-3}\,\text{y}$. The distance traveled is $8.7 \times 10^{-3}\,\text{ly}$.

<u>**73**</u>

Start with $(pc)^2 = K^2 + 2Kmc^2$, where p is the momentum of the particle, K is its kinetic energy, and m is its mass. For an electron $mc^2 = 0.511\,\text{MeV}$, so

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00\,\text{MeV})^2 + 2(2.00\,\text{MeV})(0.511\,\text{MeV})} = 2.46\,\text{MeV}\,.$$

Thus $p = 2.46\,\text{MeV}/c$.

<u>**75**</u>

The work required is the increased in the energy of the proton. The energy is given by $E = mc^2/[1-(v/c)^2]$. Let v_1 be the initial speed and v_2 be the final speed. Then the work is

$$W = \frac{mc^2}{\sqrt{1-(v_2/c)^2}} - \frac{mc^2}{\sqrt{1-(v/c)^2}} = \frac{938\,\text{MeV}}{\sqrt{1-(0.9860)^2}} - \frac{938\,\text{MeV}}{\sqrt{1-(0.9850)^2}} = 189\,\text{MeV}\,,$$

where $mc^2 = 938\,\text{MeV}$ was used.

77

(a) Let v be the speed of either satellite, relative to Earth. According to the Galilean velocity transformation equation the relative speed is $v_{\text{rel}} = 2v = 2(2.7 \times 10^4\,\text{km/h} = 5.4 \times 10^4\,\text{km/h}$.

(b) The correct relativistic transformation equation is

$$v_{\text{rel}} = \frac{2v}{1 + \frac{v^2}{c^2}} .$$

The fractional error is

$$\text{fract err} = \frac{2v - v_{\text{rel}}}{2v} = 1 - \frac{1}{1 + \frac{v^2}{c^2}} .$$

The speed of light is $1.08 \times 10^9\,\text{km/h}$, so

$$\text{fract err} = \frac{1}{1 + \frac{(2.7 \times 10^4\,\text{km/h})^2}{(1.08 \times 10^9\,\text{km/h})^2}} = 6.3 \times 10^{-10} .$$

Chapter 38

7

(a) Let R be the rate of photon emission (number of photons emitted per unit time) and let E be the energy of a single photon. Then the power output of a lamp is given by $P = RE$ if all the power goes into photon production. Now $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and $R = \lambda P/hc$. The lamp emitting light with the longer wavelength (the 700-nm lamp) emits more photons per unit time. The energy of each photon is less so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp Then

$$R = \frac{\lambda P}{hc} = \frac{(700 \times 10^{-9}\,\mathrm{m})(400\,\mathrm{J/s})}{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})(2.9979 \times 10^{8}\,\mathrm{m/s})} = 1.41 \times 10^{21}\,\mathrm{photon/s}\,.$$

17

The energy of an incident photon is $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the electromagnetic radiation, and λ is its wavelength. The kinetic energy of the most energetic electron emitted is $K_m = E - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function for sodium. The stopping potential V_0 is related to the maximum kinetic energy by $eV_0 = K_m$, so $eV_0 = (hc/\lambda) - \Phi$ and

$$\lambda = \frac{hc}{eV_0 + \Phi} = \frac{(6.626 \times 19^{-34}\,\mathrm{J \cdot s})(2.9979 \times 10^{8}\,\mathrm{m/s})}{(5.0\,\mathrm{eV} + 2.2\,\mathrm{eV})(1.602 \times 10^{-19}\,\mathrm{J/eV})} = 1.7 \times 10^{-7}\,\mathrm{m}\,.$$

Here $eV_0 = 5.0\,\mathrm{eV}$ was used.

21

(a) The kinetic energy K_m of the fastest electron emitted is given by $K_m = hf - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function of aluminum, f is the frequency of the incident radiation, and λ is its wavelength. The relationship $f = c/\lambda$ was used to obtain the second form. Thus

$$K_m = \frac{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})(2.9979 \times 10^{8}\,\mathrm{m/s})}{(200 \times 10^{-9}\,\mathrm{m})(1.602 \times 10^{-19}\,\mathrm{J/eV})} - 4.20\,\mathrm{eV} = 2.00\,\mathrm{eV}\,.$$

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_0 is given by $K_m = eV_0$, so $V_0 = K_m/e = (2.00\,\mathrm{eV})/e = 2.00\,\mathrm{V}$.

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus $hc/\lambda = \Phi$ or

$$\lambda = \frac{hc}{\Phi} = \frac{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})(2.9979 \times 10^{8}\,\mathrm{m/s})}{(4.2\,\mathrm{eV})(1.602 \times 10^{-19}\,\mathrm{J/eV})} = 2.95 \times 10^{-7}\,\mathrm{m}\,.$$

If the wavelength is longer, the photon energy is less and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

29

(a) When a photon scatters from an electron initially at rest, the change in wavelength is given by $\Delta\lambda = (h/mc)(1 - \cos\phi)$, where m is the mass of an electron and ϕ is the scattering angle. Now $h/mc = 2.43 \times 10^{-12}\,\text{m} = 2.43\,\text{pm}$, so $\Delta\lambda = (2.43\,\text{pm})(1 - \cos 30°) = 0.326\,\text{pm}$. The final wavelength is $\lambda' = \lambda + \Delta\lambda = 2.4\,\text{pm} + 0.326\,\text{pm} = 2.73\,\text{pm}$.

(b) Now $\Delta\lambda = (2.43\,\text{pm})(1 - \cos 120°) = 3.645\,\text{pm}$ and $\lambda' = 2.4\,\text{pm} + 3.645\,\text{pm} = 6.05\,\text{pm}$.

43

Since the kinetic energy K and momentum p are related by $K = p^2/2m$, the momentum of the electron is $p = \sqrt{2mK}$ and the wavelength of its matter wave is $\lambda = h/p = h/\sqrt{2mK}$. Replace K with eV, where V is the accelerating potential and e is the fundamental charge, to obtain

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2meV}} = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{C})(25.0 \times 10^{3}\,\text{V})}}\\ &= 7.75 \times 10^{-12}\,\text{m} = 7.75\,\text{pm}\,.\end{aligned}$$

47

(a) The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus $K = (1.602 \times 10^{-19}\,\text{C})(300\,\text{V}) = 4.80 \times 10^{-17}\,\text{J}$. The mass of a single sodium atom is, from Appendix F, $m = (22.9898\,\text{g/mol})/(6.02 \times 10^{23}\,\text{atom/mol}) = 3.819 \times 10^{-23}\,\text{g} = 3.819 \times 10^{-26}\,\text{kg}$. Thus the momentum of an ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819 \times 10^{-26}\,\text{kg})(4.80 \times 10^{-17}\,\text{J})} = 1.91 \times 10^{-21}\,\text{kg}\cdot\text{m/s}\,.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{1.91 \times 10^{-21}\,\text{kg}\cdot\text{m/s}} = 3.47 \times 10^{-13}\,\text{m}\,.$$

49

Since the kinetic energy K and momentum p are related by $K = p^2/2m$, the momentum of the electron is $p = \sqrt{2mK}$ and the wavelength of its matter wave is $\lambda = h/p = h/\sqrt{2mK}$. Thus

$$\begin{aligned}K &= \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{1}{2(9.11 \times 10^{-31}\,\text{kg})}\left(\frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s})}{590 \times 10^{-9}\,\text{m})}\right)^2\\ &= 6.92 \times 10^{-25}\,\text{J} = 4.33 \times 10^{-6}\,\text{eV}\,.\end{aligned}$$

59

The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$ and the wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now the kinetic energy K and the momentum are related by $K = p^2/2m$, where m is the mass of the particle. Thus $p = \sqrt{2mK}$ and

$$k = \frac{2\pi\sqrt{2mK}}{h}.$$

61

For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

Substitute $\psi = \psi_0 e^{ikx}$. The second derivative is $d^2\psi/dx^2 = -k^2\psi_0 e^{ikx} = -k^2\psi$. The result is

$$-k^2\psi + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

Solve for k and obtain

$$k = \sqrt{\frac{8\pi^2 m}{h^2}[E - U_0]} = \frac{2\pi}{h}\sqrt{2m[E - U_0]}.$$

67

(a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U and width L is given by

$$T = e^{-2kL},$$

where

$$k = \sqrt{\frac{8\pi^2 m(U - E)}{h^2}}.$$

If the change ΔU in U is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU}\Delta U = -2LT\frac{dk}{dU}\Delta U.$$

Now

$$\frac{dk}{dU} = \frac{1}{2\sqrt{U - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U - E)}\sqrt{\frac{8\pi^2 m(U - E)}{h^2}} = \frac{k}{2(U - E)}.$$

Thus

$$\Delta T = -LTk\frac{\Delta U}{U - E}.$$

For the data of Sample Problem 38–7, $2kL = 10.0$, so $kL = 5.0$ and

$$\frac{\Delta T}{T} = -kL\frac{\Delta U}{U - E} = -(5.0)\frac{(0.010)(6.8\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = -0.20.$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\,\Delta L = -2ke^{-2kL}\,\Delta L = -2kT\,\Delta L$$

and

$$\frac{\Delta T}{T} = -2k\,\Delta L = -2(6.67\times 10^{9}\,\mathrm{m}^{-1})(0.010)(750\times 10^{-12}\,\mathrm{m}) = -0.10\,.$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\,\Delta E = -2Le^{-2kL}\frac{dk}{dE}\,\Delta E = -2LT\frac{dk}{dE}\,\Delta E\,.$$

Now $dk/dE = -dk/dU = -k/2(U-E)$, so

$$\frac{\Delta T}{T} = kL\frac{\Delta E}{U-E} = (5.0)\frac{(0.010)(5.1\,\mathrm{eV})}{6.8\,\mathrm{eV}-5.1\,\mathrm{eV}} = 0.15\,.$$

There is a 15% increase in the transmission coefficient.

<u>**79**</u>

The uncertainty in the momentum is $\Delta p = m\,\Delta v = (0.50\,\mathrm{kg})(1.0\,\mathrm{m/s}) = 0.50\,\mathrm{kg\cdot m/s}$, where Δv is the uncertainty in the velocity. Solve the uncertainty relationship $\Delta x\,\Delta p \geq \hbar$ for the minimum uncertainty in the coordinate x: $\Delta x = \hbar/\Delta p = (0.60\,\mathrm{J\cdot s})/2\pi(0.50\,\mathrm{kg\cdot m/s}) = 0.19\,\mathrm{m}$.

Chapter 39

13

The probability that the electron is found in any interval is given by $P = \int |\psi|^2\,dx$, where the integral is over the interval. If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2\,\Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right),$$

so

$$P = \left(\frac{2\,\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right).$$

(a) Take $L = 100\,\text{pm}$, $x = 25\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050.$$

(b) Take $L = 100\,\text{pm}$, $x = 50\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10.$$

(c) Take $L = 100\,\text{pm}$, $x = 90\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095.$$

25

The energy levels are given by

$$E_{n_x n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right] = \frac{h^2}{8mL^2}\left[n_x^2 + \frac{n_y^2}{4}\right],$$

where the substitutions $L_x = L$ and $L_y = 2L$ were made. In units of $h^2/8mL^2$, the energy levels are given by $n_x^2 + n_y^2/4$. The lowest five levels are $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, $E_{2,1} = 4.25$, and $E_{2,2} = E_{1,4} = 5.00$. A little thought should convince you that there are no other possible values for the energy less than 5.

The frequency of the light emitted or absorbed when the electron goes from an initial state i to a final state f is $f = (E_f - E_i)/h$ and in units of $h/8mL^2$ is simply the difference in the values of $n_x^2 + n_y^2/4$ for the two states. The possible frequencies are 0.75 (1,2 $\rightarrow$ 1,1), 2.00 (1,3 $\rightarrow$ 1,1), 3.00 (2,1 $\rightarrow$ 1,1), 3.75 (2,2 $\rightarrow$ 1,1), 1.25 (1,3 $\rightarrow$ 1,2), 2.25 (2,1 $\rightarrow$ 1,2), 3.00 (2,2 $\rightarrow$ 1,2), 1.00 (2,1 $\rightarrow$ 1,3), 1.75 (2,2 $\rightarrow$ 1,3), 0.75 (2,2 $\rightarrow$ 2,1), all in units of $h/8mL^2$.

There are 8 different frequencies in all. In units of $h/8mL^2$ the lowest is 0.75, the second lowest is 1.00, and the third lowest is 1.25. The highest is 3.75, the second highest is 3.00, and the third highest is 2.25.

33

If kinetic energy is not conserved some of the neutron's initial kinetic energy is used to excite the hydrogen atom. The least energy that the hydrogen atom can accept is the difference between the first excited state ($n = 2$) and the ground state ($n = 1$). Since the energy of a state with principal quantum number n is $-(13.6\,\text{eV})/n^2$, the smallest excitation energy is $13.6\,\text{eV} - (13.6\,\text{eV})/(2)^2 = 10.2\,\text{eV}$. The neutron does not have sufficient kinetic energy to excite the hydrogen atom, so the hydrogen atom is left in its ground state and all the initial kinetic energy of the neutron ends up as the final kinetic energies of the neutron and atom. The collision must be elastic.

37

The energy E of the photon emitted when a hydrogen atom jumps from a state with principal quantum number u to a state with principal quantum number ℓ is given by

$$E = A\left(\frac{1}{\ell^2} - \frac{1}{u^2}\right),$$

where $A = 13.6\,\text{eV}$. The frequency f of the electromagnetic wave is given by $f = E/h$ and the wavelength is given by $\lambda = c/f$. Thus

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E}{hc} = \frac{A}{hc}\left(\frac{1}{\ell^2} - \frac{1}{u^2}\right).$$

The shortest wavelength occurs at the series limit, for which $u = \infty$. For the Balmer series, $\ell = 2$ and the shortest wavelength is $\lambda_B = 4hc/A$. For the Lyman series, $\ell = 1$ and the shortest wavelength is $\lambda_L = hc/A$. The ratio is $\lambda_B/\lambda_L = 4$.

43

The proposed wave function is

$$\psi = \frac{1}{\sqrt{\pi}\,a^{3/2}} e^{-r/a},$$

where a is the Bohr radius. Substitute this into the right side of Schrödinger's equation and show that the result is zero. The derivative is

$$\frac{d\psi}{dr} = -\frac{1}{\sqrt{\pi}\,a^{5/2}} e^{-r/a},$$

so

$$r^2\frac{d\psi}{dr} = -\frac{r^2}{\sqrt{\pi}\,a^{5/2}} e^{-r/a}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{1}{\sqrt{\pi}\,a^{5/2}}\left[-\frac{2}{r} + \frac{1}{a}\right]e^{-r/a} = \frac{1}{a}\left[-\frac{2}{r} + \frac{1}{a}\right]\psi.$$

Now the energy of the ground state is given by $E = -me^4/8\epsilon_0^2 h^2$ and the Bohr radius is given by $a = h^2\epsilon_0/\pi m e^2$, so $E = -e^2/8\pi\epsilon_0 a$. The potential energy is given by $U = -e^2/4\pi\epsilon_0 r$, so

$$\frac{8\pi^2 m}{h^2}[E - U]\psi = \frac{8\pi^2 m}{h^2}\left[-\frac{e^2}{8\pi\epsilon_0 a} + \frac{e^2}{4\pi\epsilon_0 r}\right]\psi = \frac{8\pi^2 m}{h^2}\frac{e^2}{8\pi\epsilon_0}\left[-\frac{1}{a} + \frac{2}{r}\right]\psi$$
$$= \frac{\pi m e^2}{h^2\epsilon_0}\left[-\frac{1}{a} + \frac{2}{r}\right]\psi = \frac{1}{a}\left[-\frac{1}{a} + \frac{2}{r}\right]\psi .$$

The two terms in Schrödinger's equation obviously cancel and the proposed function ψ satisfies that equation.

47

The radial probability function for the ground state of hydrogen is $P(r) = (4r^2/a^3)e^{-2r/a}$, where a is the Bohr radius. (See Eq. 39–44.) You want to evaluate the integral $\int_0^\infty P(r)\,dr$. Eq. 15 in the integral table of Appendix E is an integral of this form. Set $n = 2$ and replace a in the given formula with $2/a$ and x with r. Then

$$\int_0^\infty P(r)\,dr = \frac{4}{a^3}\int_0^\infty r^2 e^{-2r/a}\,dr = \frac{4}{a^3}\,\frac{2}{(2/a)^3} = 1 .$$

49

(a) ψ_{210} is real. Simply square it to obtain the probability density:

$$|\psi_{210}|^2 = \frac{r^2}{32\pi a^5}e^{-r/a}\cos^2\theta .$$

(b) Each of the other functions is multiplied by its complex conjugate, obtained by replacing i with $-i$ in the function. Since $e^{i\phi}e^{-i\phi} = e^0 = 1$, the result is the square of the function without the exponential factor:

$$|\psi_{21+1}|^2 = \frac{r^2}{64\pi a^5}e^{-r/a}\sin^2\theta$$
$$|\psi_{21-1}|^2 = \frac{r^2}{64\pi a^5}e^{-r/a}\sin^2\theta .$$

The last two functions lead to the same probability density.

(c) For $m_\ell = 0$ the radial probability density decreases strongly with distance from the nucleus, is greatest along the z axis, and for a given distance from the nucleus decreases in proportion to $\cos^2\theta$ for points away from the z axis. This is consistent with the dot plot of Fig. 39–24 (a). For $m_\ell = \pm 1$ the radial probability density decreases strongly with distance from the nucleus, is greatest in the x, y plane, and for a given distance from the nucleus decreases in proportion to $\sin^2\theta$ for points away from that plane. Thus it is consistent with the dot plot of Fig. 39-24(b).

(d) The total probability density for the three states is the sum:

$$|\psi_{210}|^2 + |\psi_{21+1}|^2 + |\psi_{21-1}|^2 = \frac{r^2}{32\pi a^5}e^{-r/a}\left[\cos^2\theta + \frac{1}{2}\sin^2\theta + \frac{1}{2}\sin^2\theta\right]$$
$$= \frac{r^2}{32\pi a^5}e^{-r/a} .$$

The trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ was used. The total probability density does not depend on θ or ϕ. It is spherically symmetric.

<u>57</u>

The wave function is $\psi = \sqrt{C}e^{-kx}$. Substitute this function into Schrödinger's equation,

$$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2} + U_0\psi = E\psi .$$

Since $d^2\psi/dx^2 = \sqrt{C}k^2e^{-kx} = k^2\psi$, the result is

$$\frac{h^2k^2}{8\pi^2 m}\psi + U_0\psi = E\psi .$$

The solution for k is

$$k = \sqrt{\frac{8\pi^2 m}{h^2}(U_0 - E)} .$$

Thus the function given for ψ is a solution to Schrödinger's equation provided k has the value calculated from the expression given above.